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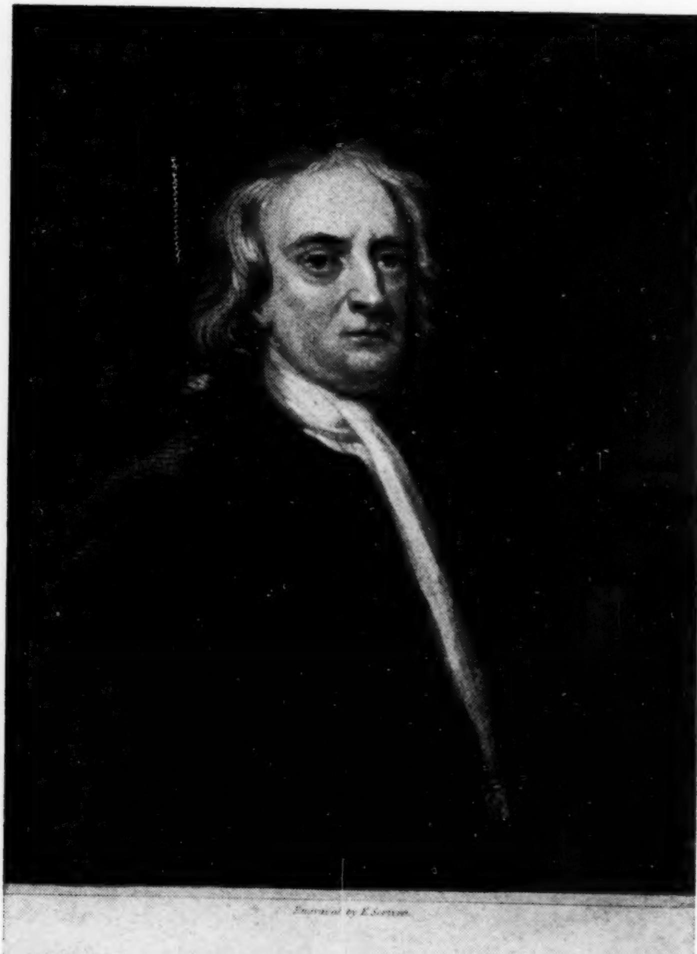

SIR ISAAC NEWTON.

THE MATHEMATICS TEACHER

Volume XXVI Number 2

Edited by William David Reeve

Mathematical Induction

By Raymond Garver
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The student in college algebra does not ordinarily have the opportunity to see very clearly the importance and wide applicability of proofs by mathematical induction. He uses the method in summing certain series to n terms, and in the proof of the binomial theorem for positive integral exponents, and then drops it, usually with a sigh of relief. Consequently in a recent more advanced course in algebra I thought it worth while to devote some little time to a study of various applications of the method. The list of applications which was thus obtained may, I hope, be of sufficient interest to warrant its presentation to readers of The Mathematics Teacher. Of course, the list is obviously not complete; there are many examples from higher mathematics which I did not desire to include. I wanted only problems which the students could work through. I should say that in the class presentation I used none of the references to Dickson's *Modern Algebraic Theories*, and not all of those to Bôcher's *Higher Algebra*. However, these are standard sources, and some readers may be interested in the use they make of induction.

I. Applications in Elementary Algebra.

1. Summing of certain series to n terms. Examples may be found in any college algebra text.

2. Proof of the binomial theorem for positive integral exponents.
3. Algebraic divisibility theorems. $x^n - y^n$ is divisible by $x - y$ when n is a positive integer; $x^n - y^n$ and $x^n + y^n$ are divisible by $x+y$ under certain conditions, but not otherwise; $x^n + y^n$ is not divisible by $x - y$. The first of these may be used to prove the factor theorem; see Wilczynski and Slaught, *College Algebra,* 1916, p. 132.

II. Applications in the Mathematics of Finance.

1. The rigorous derivation of the compound interest law involves an inductive step.
2. The usual formulas for the present value and for the amount of an annuity can be proved very nicely by induction.

III. Applications in Trigonometry.

1. The sums of a number of trigonometric series can be verified easily by induction, for example,
$\sin x + \sin 3x + \ldots\ldots + \sin (2n-1)x = \sin^2 nx / \sin x,$
$\cos x + \cos 3x + \ldots\ldots + \cos (2n-1)x = \sin 2nx / 2 \sin x.$
These are special cases of two well known series which arise in certain problems in electricity; see Palmer and Leigh, Plane and Spherical Trigonometry, 3rd ed., 1925, p. 120.
2. The inductive principle may be used in the derivation of the ordinary addition formulas for sin $(x + y)$ and cos $(x+y)$; see Wilczynski and Slaught, *Plane Trigonometry,* 1914, pp. 169-70, 180-4. The induction here is from Θ to $\Theta + 90$, rather than from n to $n + 1$.
3. More general addition formulas for the sine and cosine of the sum of n angles are established inductively by Hobson in his *Plane Trigonometry,* 4th ed., 1918, pp. 48-9. He also obtains, on pp. 50-1, formulas for the product of the sines (or cosines) of n angles as a sum of sines or cosines of composite angles.

IV. Applications in the Theory of Equations.

1. De Moivre's Theorem. See Dickson, *First Course in the Theory of Equations,* 1922, p. 5.
2. Relations between the roots and coefficients of an equation. See Dickson, p. 18.

3. If $f(x)$ is a polynomial of the nth degree, $f(x) \equiv a_0x^n + a_1x^{n-1} + \ldots\ldots + a_n$, there exists one and only one set of constants $\alpha_1\ \alpha_2\ \ldots\ldots\ \alpha_n$, such that $f(x) \equiv a_0(x-\alpha_1)(x-\alpha_2)\ldots\ldots(x-\alpha_n)$. See Bôcher, *Higher Algebra*, 1919, p. 17. The proof also uses the factor theorem and fundamental theorem of algebra.
4. The fundamental theorem on symmetric functions. See Weber and Wellstein, *Encyklopädie der Elementar-Mathematik*, vol. 1, 3rd ed., 1909, p. 228.

V. Applications in the Calculus.

1. The theorems on the limit of a sum and the limit of a product may be extended from 2 to n variables by induction. It may also be mentioned that probability may be defined in such a way that some of the important theorems in the theory of probability may be proved similarly. See Coolidge, *Mathematical Probability*, 1925, pp. 17-18.
2. The formula for the first derivative of a product of n functions of the independent variable.
3. Leibniz' rule for the nth derivative of a product of two functions.
4. Formulas for the nth derivatives of a number of functions, as $\log x$, $\sin ax$, $\cos ax$, $x \sin x$, xe^{2x}, $\cos^2 x$, may be proved true by induction. One more difficult result of this type is $\dfrac{d^n}{dx^n}(e^x \cos x) = 2^{n/2} e^x \cos\left(x + \dfrac{n\pi}{4}\right)$. For specific formulas and other results the reader may see Osborne, *Calculus*, 1891, pp. 42-5; Johnson, *Differential Calculus*, 1904, pp. 103-4.

VI. Applications in the Theory of Numbers.

1. Arithmetic divisibility theorems. $3^{2n+2} - 8n - 9$ (n a positive integer) is a multiple of 64; if n is odd, $n(n^2-1)$ is a multiple of 24; $A^{2n+1} + (A-1)^{n+2}$ is a multiple of $A^2 - A + 1$, and so on. Many results of this kind may be found in Hall and Knight's *Algebra* and in C. Smith's *Treatise on Algebra*.
2. The product of n consecutive positive integers is divisible by n! See Hall and Knight, *Higher Algebra*, 4th ed., 1922, p. 345.

3. Fermat's theorem—If p is a prime, $n^p - n$ is divisible by p. See Hall and Knight, p. 356; Weber and Wellstein, p. 194.
4. The proof of the general formula for the Euler Φ-function may be done inductively. See Carmichael, *Theory of Numbers,* 1914, pp. 33-4.
5. The well-known formulas for the number of divisors of a composite number and for the sum of these divisors may be proved quite easily by induction. I have not seen such proofs in print.

VII. Applications in Higher Algebra.

1. The multinomial theorem for positive integral exponents. See Weber and Wellstein, vol. 1, p. 195.
2. Theorems on polynomials in several variables. See Bôcher, pp. 5, 6, 10, 11.
3. The formula for the nth term of an arithmetic progression of higher order. The proof of this formula is noteworthy because, in the inductive step, a double use of the assumption that the formula is true for n is required to prove its truth for $(n+1)$. See Wilson and Warren, *College Algebra,* 1928, p. 385.
4. Theorems on matrices, especially symmetric matrices. See Bôcher, pp. 57, 65; Dickson, *Modern Algebraic Theories,* 1926, pp. 80, 82.
5. Hilbert's theorem on binary forms. See Dickson, p. 30.
6. A part of the theorem on the finiteness of a fundamental system of invariants may be established by induction. See Dickson, p. 32.
7. The canonical form of a binary form of odd order. See Dickson, p. 36.
8. Induction is used in a part of the theorem that an algebraic equation having a solvable group for a field F containing its coefficients is solvable by radicals relatively to F. See Dickson, p. 193.
9. The arithmetic mean of n positive quantities is not less than their geometric mean. See Chrystal, *Algebra,* 2nd ed., 1926, p. 46.
10. Several of the fundamental properties of continued fractions are established inductively. See Chrystal, p. 432, 435.
11. The sum of the products of the integers from 1 to k taken

two at a time is equal to $(k-1)k(k+1)(3k+2)/24$. The sum of the squares of these products is equal to $(k-1)k(k+1)(4k^2-1)(5k+6)/360$. These results are rather more difficult to obtain than those usually taken up in elementary algebra.

VIII. Miscellaneous Applications.

1. The sum of the angles of a closed convex polygon of n sides is $(n-2)$ times two right angles.
2. An interesting geometrical example of mathematical induction may be found in the *American Mathematical Monthly*, vol. 34, 1927, pp. 247-50. A geometrical proof that $\sin x/x$ continually decreases as x increases from O to $\pi/2$, and of several related theorems, is given.
3. A few general determinants may be evaluated with the aid of induction. For an example, see the *American Mathematical Monthly*, vol. 38, 1931, p. 355.
4. The nth term of the series, 1, 3, 4, 7, . . . where each term after the second is the sum of the two which precede it, is equal to $\frac{1}{2^n}[1+\sqrt{5})^n+(1-\sqrt{5})^n]$. This proof is interesting because, in the inductive step, it is necessary to assume the truth of the formula for two consecutive values of n to prove its truth for the following value.
5. Certain fundamental properties of finite aggregates may be proved by aggregates. See Hobson, *Theory of Functions of a Real Variable*, 2nd ed., vol. 1, 1921, pp. 4-5. These proofs might not be out of place in a class which was making a serious study of the real number system.

Hobson's proof of the validity of mathematical induction, given on page 7, may also be of interest. For a logical criticism of the method of induction the reader may see Bell, "On Proofs by Mathematical Induction," *American Mathematical Monthly*, vol. 27, 1920, pp. 413-15. A fairly good knowledge of certain recent developments will be necessary to fully understand the point of this note. An interesting historical article on induction, by Bussey, may be found in the American Mathematical Monthly, vol. 24, 1917, pp. 199-207. Professor Bussey also gives a number of modern applications of induction; my applications V, 1, and V, 4, were borrowed from his list. Most of his other applications are also given in my list.

High School Mathematics Club*

FELLOW STUDENTS OF MATHEMATICS: A wise old observer has remarked that there is as much difference in folks as there is in anybody. Take the cases of A and B. These impersonal names are used for typical members of any mathematics class. A (Alice or Arthur) comes to the teacher with the request "Can't you find us some harder originals? Even father could do all that you assigned for last night." B (Betty or Bernard) closes the book just before recitation and sighs audibly "There, if he calls on me early and lets me alone, I can prove his parallelograms equal but, if he makes me stop to give reasons, I'm sunk." Scattered between A and B are the other members of the class, not so ambitious as to want to do much more than was called for, and not so stupid as to believe that mathematics can be mastered by memorizing a textbook. This talk is an attempt to show that a well-conducted mathematics club will have something of value for all kinds of pupils. We may state and explore our purpose as a
Theorem:

If a mathematics club is alive and alert,
then it will be of value to all members of the class.

When the formation of a club is proposed there will be opposition. Very little of it will come from pupils like A who would be doing interesting mathematical things for themselves, club or no club. These enthusiasts will get so keen a pleasure out of the meetings that they will be willing to put time and vital energy into making the club a success. Let us pause to look at the word "enthusiast"; it is from a Greek word meaning inspired, God-possessed, and still has that as its best meaning but it is also sometimes used as a polite way of saying "a little bit crazy." Be enthusiasts in the best sense and pay as little attention as you can to outsiders who sneer "Huh, as if anybody who wasn't a bit crazy could ever get enthusiastic about mathematics." Unfortunately, the pupil we have called B will be found among the grousers saying "Classes were not enough; they have to go sticking in this club with some dry old program once a month. I suppose teacher will expect us to go. Anyhow, I'm going once and I hope they limber up and stand treat at that meeting." The remedy is to avoid dry old

* A radio talk by Norman Anning, University of Michigan, over WJR, 2:00 P.M., Oct. 20, 1932. A bibliography to accompany this article is given on page 75.

programs. Into every meeting try to put fun, variety, competition, informality. It was said by P. E. B. Jourdain that "thinking logically is closely allied to seeing a joke." Mathematics and fun are not to be divorced. You can see a joke (fortunately for the speaker television has not yet been perfected). Try this one: "Why is simplifying a fraction like powdering the nose?" The answer is, "Because it improves the appearance without changing the value."

Let me give some hints about organization. There will of course be a faculty adviser. She should be like the spare tire, inconspicuous but ready when needed. The simile is a poor one because the tire ought to be full of wind; the teacher in this situation ought to be silent or at least be heard as little as possible. The choice of officers, name, and constitution should come from the members. Without external stimulus the president will preside, the secretary will record, and committees will "commit" as efficiently as the average corresponding group of adults. Indeed, the student group that cannot generate sufficient energy for this part of its activities cannot long survive and will soon not need any faculty advice. You may observe that the end of that sentence has the rhythm of the dead march. May I suggest that the members establish the rotating office of critic and select from their number a different critic for each meeting. This critic (in the older sense of the word) should take his task seriously and report at the end of the meeting bestowing praise and blame. The fact that he is paying careful attention will make the others watch their steps. Critical suggestions from the faculty adviser should be given in private to individual members and, if it is possible, only when asked for.

After organization the next natural question is, "What can we do?" One good thing would be to start a club library. If you go about it correctly I am sure the school library and the nearest public library will be glad to help you. Into your library you ought to put histories like those of Ball, Cajori, Heath, Sanford, Smith. If you can buy only one, start with Miss Sanford's. The history of arithmetic by Karpinski in rich in pictures. So also is the two-volume work of David Eugene Smith. Sir Thomas Heath wrote two excellent volumes on Greek mathematics but the gist of it has now been condensed into a single book. If your school library has the new Britannica, many a topic for the club can be traced in it. For instance, if I were to be your next speaker, I could not ask for a better source of ideas than the article on special curves by Archibald. Or one might look under weaving and bring back a paper on the geometry of satin patterns.

On the philosophical side you might start with Bell's *Queen of the Sciences* and add to it books by Keyser, Russell, Shaw. Bertrand Russell's book was written during the late unpleasantness. Because his conscience would not let him fight he was kept a prisoner by the British government. But they could not imprison his wit. The following quotation is a sample: ". . . if, instead of supposing x to be a man, we were to suppose him to be a monkey or a goose or a Prime Minister." Professor Keyser of Columbia tells in his book, *Pastures of Wonder,* what mathematics is and what it does. This book and the one by Bell are rich in ammunition for the person who undertakes to defend mathematics from the attacks of those who do not understand it.

You ought to have Moritz's book of quotations and Alfred Noyes's poem, "Watchers of the Sky." Smith's *Source Book* will permit you to listen to mathematical giants explaining their discoveries in their own words. The words have, where necessary, been translated into English but you must not expect everything to be translated for you. Nearly all of you are in language classes. What could be more interesting than a few textbooks used by boys and girls of your own age in France, or Germany, or Italy?

You will want plenty of books of puzzles and entertainment. A dozen excellent ones could be listed. Let's be satisfied with three: Dudeney's *Canterbury Puzzles,* Jones's older book, *Mathematical Wrinkles,* and his newer one, *Mathematical Nuts.* Do not misunderstand the latter title; they are nuts to be cracked.

When your turn comes to put on an assembly program you will perhaps look around for a play. There are not many because mathematics does not readily lend itself to dramatization. One might start off bravely with the title, "The tragedy of an angular figure," but where would you go from there? Some of the best available plays can be found in the files of THE MATHEMATICS TEACHER for the last ten years. A few others are in *School Science and Mathematics* or in the *American Mathematical Monthly.* The last-named journal contains a department which will give you a great deal of information about effective programs that have been used by other clubs.

School Science and Mathematics has a problem department. If you polish up and send in a solution of some proposed problem, your club will be given the credit. I suspect you are human enough to be pleased when you find your name in print.

There is a national mathematical fraternity, Pi Mu Epsilon. Of course, no high school club could seek affiliation with it and no per-

son who is not a member could do more than guess at the meaning of the letters. Let us agree for this afternoon that P.M.E. means PUSH MATHEMATICS ENERGETICALLY. That is a slogan we can all adopt without fear that the fraternity will quarrel with us for stealing their fire.

Some day this request may come to your adviser: "Miss Wilkins, can your club fill that display cabinet in the main hall?" The answer ought to be "We surely can." Every club should start a collection of models that have been made and demonstrated by its members. No club is so small or so poor that it cannot enjoy the possession of some pictures, graphs, charts, designs, diagrams, maps, mosaics. I do not refer to a heterogeneous omnium gatherum but to material which has been used in the club and in which individual members have taken an active interest. I recently saw in a school exhibit a booklet that had been made by an eighth grader. The title was Plato's aphorism, "God geometrizes continually." After a neatly-written paragraph about Plato, the book was filled with pictures of geometrical forms which appear in nature. The idea was excellent and, while one may doubt whether a robin is a combination of a cone, a sphere, and a cylinder, no one can doubt that the pupil who made that book is now looking on nature with a keener appreciation of form. If you are tempted to prepare a book to show that man geometrizes intermittently, be careful about the sources of your illustrations. It is true and it is also trivial that a certain nourishing soup is delivered to the public in right circular cylinders.

Let me say again that the most valuable models are those which you think out and construct with your own hands in your own shop. In making the five regular solids you will gain some skill in managing knife, cardboard, and gummed paper. Go right ahead and try to make one of the star solids. Did you know that a working pantograph for enlarging or contracting drawings could be made out of four pieces of Erector steel suitably jointed? Or that a simpler pantograph could be made out of a piece of elastic with three marks on it? Figure it out for yourself and use it when your class comes to the study of similar triangles. It is easy to saw a cube of wood into pieces in such a way that you can prove to anybody that the difference of two cubes can always be factored. Here is one for the girls: How did Betsy Ross fold the cloth if she could cut out a perfect five-pointed star with a single snip of the shears? If there is a blacksmith among you here is a problem for him: How can you fasten three links together in such a way that all can be separated by cutting open any one? It can be done. And a hole can be cut through a cube so that a bigger cube can

be shoved through. Your club could spend more than one pleasant evening studying dissection proofs of the theorem of Pythagoras. Many good ones are known but you might find a better.

Do not be ashamed to ask assistance outside of the school or outside of the domain usually called mathematics. There must be someone in your school or in your town who will be proud to be asked to come and show you how to multiply, to divide, and to extract square roots on the adding machine. Have an engineer bring his transit and level and show how they do the job for which they were designed. While you have him, find out why his slide rule is called a "guessing stick" and how he knows where to put the decimal point after he has finished a computation. Perhaps you can get a Chinaman to come in and do problems for you on the swan-pan. And you ought to be pleased to see some of the things a master carpenter can do with a steel square. When you need a stretching exercise, invite an astronomer. After some friendly chemist has explained a model of the atom, that quotation from Plato will take on a new meaning for you. Watch for the ramifications of our subject. For instance, Lavoisier's proof that air is two things and not one is pure mathematics. And in the region of philosophy Spinoza attempted to give a mathematical demonstration of the proposition: *That God is.*

But you will say that you cannot always be entertained by strangers. That is true but there are dozens of good ways in which you can entertain and instruct yourselves. Try a dictionary evening. What mathematical word sounds as if the parrot was lost? Correct, polygon. A pleasant trip to the dictionary will show you that polygon means "many corners," diagonal means "through the corners," diameter means "measure across," and eliminate means "out over the doorstep." The pupil who has traced the language connection between "hypothesis" and "supposition" will never again (or hardly ever) confuse hypothesis with hypotenuse.

Another good topic is the breakdown of the rule of three. If a pair of scissors is worth forty cents, what is a fair price for a scissor? If a jeweler can put a crystal on my watch in ten minutes, how long would it take a hundred jewelers? If a woman can boil an egg in three minutes, how many women would it take to boil an egg in one minute? If a boy can mow the lawn in an hour, would six boys mow it in ten minutes?

If you believe that a pun is a crime, you might have a delightful half-hour and seek absolution for a lot of crimes at once by sweeping

all the current mathematical jokes into a single piece of black-face comedy:

Mistuh Deacon, can you spell rectangular parallelopiped, suh, in three letters?

Yes, ah kin, Mistuh Cawnelius, B-O-X. And I'se askin' you what is the shape of a kiss?

That is easy, Mr. Deacon. A kiss is a lip tickle.

What is a curve?

A curve is when you can see the locomotive from the last car.

Occasionally for the sake of variety and to encourage the shy ones you might have a symposium to which every member must contribute a little bit. Again, you might invite a sister club and run one of those double-barrelled programs like "the poetry of motion and the motion of poetry" or "the mathematics of music and the music of mathematics." There is music in mathematics; even so prosaic a thing as the formula for solving a quadratic equation can be set to something resembling music.

When you have to serve refreshments, there are cones that can be intersected, delicious cubes that float because their density is less than that of the circumfluent liquid, and doughnuts which any good cook can generate by revolving a circle about a line in its plane.

Finally, one thing not to do. Do not look for hard topics. Keep away from angle-trisection, time-binding, relativity. And here are some things every member should do. You can, whether your name is A or B or any other letter down to ampersand, be regular in attendance, be willing to help on committees, keep alert enough to ask questions of the speaker, and be always figuring out something for yourself which you hope to polish up and present at some future meeting of your club.

Our theorem of page 70 ~~so and so~~ has not been proved but something has been done to make it appear plausible.

I thank you.

UNIVERSITY OF MICHIGAN
LIBRARY EXTENSION SERVICE

BOOKS FOR A HIGH SCHOOL MATHEMATICS CLUB LIBRARY

Archibald, Raymond C. Outline of the history of mathematics. 1932. 53p. 30c

Ball, W. W. R. A short account of the history of mathematics. N.Y., Macmillan, 1915. 536p. $4

Becker, H., and Vonderlinn, I. Geometrisches Zeichnen. Berlin, Walter de Gruyter, 1923. 136p. 40c (Sammlung Goeschen No. 58)

Bell, Eric T. The queen of the sciences. Baltimore, Williams and Wilkins, 1931. 138p. illus. $1

Boon, Fred C. Companion to elementary school mathematics. N.Y., Longmans, 1924. 321p. $4.75

Cajori, Florian A. A history of mathematics. N.Y., Macmillan, 1919. 514p. $4

Clifford, William K. Common sense of the exact sciences. N.Y., Appleton, 1885. 271p. illus. out of print

Dudeney, Henry E. Canterbury puzzles and other curious problems. N.Y., Nelson, 1907. 194p. $1.50

Dudeney, Henry E. Amusements in mathematics. N.Y., Nelson, 1917. 258p. $1.50

Gheury de Bray, M. E. J. Exponentials made easy. N.Y., Macmillan, 1927. 253p. $1.75

Heath, Sir Thomas L. Manual of Greek mathematics. N.Y., Oxford University Press, 1931. 552p. $5

Jones, Samuel I. Mathematical wrinkles, 4th ed. The author, Life and Casualty Bldg., Nashville, Tenn., 1929. 361p. illus. $3

Jones, Samuel I. Mathematical nuts for lovers of mathematics. The author, Life and Casualty Bldg., 1932. 240p. illus. $3.50

Jourdain, Philip E. B. The nature of mathematics. London, T. C. Jack, 1913. 92p. out of print

Karpinski, Louis C., History of arithmetic. Chicago, Rand, 1925. 200p. $2

Keyser, Cassius J. Mathematical philosophy. N.Y., Dutton, 1922. 466p. $3

Keyser, Cassius J. Pastures of wonder. N.Y., Columbia University Press, 1929. 208p. $2.75

Kraitchik, M. La mathematique des jeux, ou recreations mathematiques. 1930. Paris, Gauthier-Villars.

Laisant, Charles A. Mathematics. London, Constable, 1913. 158p. illus. out of print

Lietzmann, Walther. Lustiges und merkwurdiges von zahlen und formen. Breslau, F. Hirt, 1930. 195p. 9.50 marks

Moritz, Robert C. Memorabilia mathematica. N.Y., Macmillan, 1914. 410p. $4

Noyes, Alfred. Watchers of the sky. N.Y., Stokes, 1922. 281p. $2.50 (Torchbearers, Vol. I)

Russell, Bertrand. Introduction to mathematical philosophy. N.Y., Macmillan, 1919. 200p. $4

Sanford, Vera. A short history of mathematics. Boston, Houghton, 1930. 402p. illus. $3.25

Smith, David E. A history of mathematics. Boston, Ginn, 1920. 2v. illus. v.1—$4; v.2—$4.40

Smith, David E. Source book in mathematics. N.Y., McGraw-Hill, 1929. 710p. illus. $3

ON TO MINNEAPOLIS!

Practical Mathematics of Roman Times

By MARGARET FIELDS
Mineola, Long Island, New York

FEW PROFESSIONAL WRITINGS have exerted so great an influence on the practitioners of art as the *De Architectura* of Vitruvius (*c.* 20 B.C.). Upon the invention of printing it was one of the first books to be published, three editions being included among the incunabula, and during the Renaissance it was the handbook of all the great architects and engineers.

The *De Architectura* becomes immediately of interest to the mathematician when, in his introduction, Vitruvius writes as follows: "Geometry, also, is of much assistance in architecture, and in particular it teaches us the use of the rule and compasses, by which especially we acquire readiness in making plans for buildings in their grounds, and rightly apply the square, the level and the plummet. By means of optics, again, the light in buildings can be drawn from fixed quarters of the sky. It is true that it is by arithmetic that the total cost of buildings is calculated and measurements are computed, but difficult questions involving symmetry are solved by means of geometrical theories and methods."

It being impossible to give here an exhaustive account of the many topics treated by Vitruvius, only those which seem to be of greatest interest are presented.

Requirements of Architecture

Architecture depends on Order, Arrangement, Eurythmy, Symmetry, Propriety, and Economy. Of these, *Order* concerns symmetrical agreement of the proportions of the whole. *Arrangement* has to do with forms of expression such as groundplan, elevation and perspective. *Symmetry* considers the proper agreement between different parts and the whole general scheme. In the human body there is symmetrical harmony between the forearm, foot, palm and finger—"so it is with perfect buildings."

On Symmetry: In Temples and in the Human Body

The design of a temple depends on symmetry which is due to proportion. Proportion is a correspondence among the measures of the

members of an entire work to a certain part selected as standard. From this result the principles of symmetry. Without symmetry there can be no principles in the design of any temple; that is, if there is no precise relation between its members as in the case of those of a well-shaped man. The human body is so designed that the face from the chin to the top of the forehead is a tenth part of the whole height; the open hand from the wrist to the tip of the middle finger is just as the same etc. Other members, too, have symmetrical proportions and it was by employing them that the famous painters and sculptors of antiquity attained to great and endless renown. Thus there is good reason for the rule that the different members of a perfect building must be in exact symmetrical relation to the whole general scheme.

In the actual building of the temple the columns should be placed so that there are twice as many intercolumniations on the sides as in front, thus the length will be twice the width. The steps in front should be arranged so that there is always an odd number; thus the right foot with which one mounts the first will be the first to reach the level of the temple. The columns at the corners should be made thicker than the others by one fiftieth of their own diameter because they are sharply outlined by unobstructed air and seem to be more slender than they are. Hence, we must counteract the ocular deception by an adjustment of proportions.

Ionic Architecture

All parts above the capitals of the columns should be inclined to the front a twelfth part of their own height for when we stand in front of them, if two lines are drawn from the eye, one reaching to the top of the building and the other to the bottom, the one from the upper part will be longer and make it look as if it were leaning back, but when inclined to the front, they will appear to be plumb.

Doric Columns

The Doric columns should be fluted with twenty flutes. If these are to be channeled out, the contour of the channeling may be determined thus: Draw a square with sides equal in length to the breadth of the fluting, and center a pair of compasses in the middle of this square. Then describe a circle with a circumference touching the angles of the square, and let the channelings have the contour of the segment formed by the circumference and the side of the square.

Directions of the Streets

In laying out a town, streets must be laid with due regard to climatic conditions; cold winds are disagreeable, hot winds are enervating and moist winds are unhealthful. Therefore, by shutting out winds from our dwellings, we make them healthful. The method of procedure is as follows: In the middle of the city let a spot be made "true" by means of the rule and level. In the center of this spot set up a bronze gnomon or "shadow tracker." At about the fifth hour of the morning mark the end of the shadow cast by the gnomon. Then opening your compasses to this point —*B*— describe a circle from the center. In the afternoon when the shadow once more touches the circumference of this circle, mark it with a point *C*. From these two points describe intersecting arcs, and through their intersection and the center let a line —*DE*— be drawn to the circumference of the circle to give us the quarters of north and south. Then using a sixteenth part of the circumference of the circle as a diameter, describe a circle with its center on the line to the south, at the point where it crosses the circumference, and put points to the right and left on the circumference on the south side, repeating the process on the north side. From the four points thus obtained draw lines intersecting the center and from one side of the circumference to the other. Thus we shall have an eighth part of the circumference. The rest of the circumference is then to be divided into three equal parts on each side, and thus we have designed a figure equally apportioned among the eight winds. Then let the direction of the streets and alleys be laid down on the lines of division between the quarters of two winds.

Leveling and Leveling Instruments

Vitruvius considers three types of leveling instruments—dioptrae, water levels, and chorobates. However, he considers the first two as being very deceptive and describes the chorobate only. It is a straightedge about twenty feet long. At the extremities it has legs, made exactly alike and jointed on perpendicularly to the extremities of the straightedge, and also crosspieces fastened by tenons connecting the straightedge and the legs. These crosspieces have vertical lines drawn upon them, and there are plumblines hanging from the straightedge over each of the lines. When the straightedge is in position and the plumblines strike both the lines alike and at the same time, the instrument is level. If the wind interposes, have a groove on the upper side

members of an entire work to a certain part selected as standard. From this result the principles of symmetry. Without symmetry there can be no principles in the design of any temple; that is, if there is no precise relation between its members as in the case of those of a well-shaped man. The human body is so designed that the face from the chin to the top of the forehead is a tenth part of the whole height; the open hand from the wrist to the tip of the middle finger is just as the same etc. Other members, too, have symmetrical proportions and it was by employing them that the famous painters and sculptors of antiquity attained to great and endless renown. Thus there is good reason for the rule that the different members of a perfect building must be in exact symmetrical relation to the whole general scheme.

In the actual building of the temple the columns should be placed so that there are twice as many intercolumniations on the sides as in front, thus the length will be twice the width. The steps in front should be arranged so that there is always an odd number; thus the right foot with which one mounts the first will be the first to reach the level of the temple. The columns at the corners should be made thicker than the others by one fiftieth of their own diameter because they are sharply outlined by unobstructed air and seem to be more slender than they are. Hence, we must counteract the ocular deception by an adjustment of proportions.

Ionic Architecture

All parts above the capitals of the columns should be inclined to the front a twelfth part of their own height for when we stand in front of them, if two lines are drawn from the eye, one reaching to the top of the building and the other to the bottom, the one from the upper part will be longer and make it look as if it were leaning back, but when inclined to the front, they will appear to be plumb.

Doric Columns

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5 feet long and one digit wide and a digit and a half deep and pour water into it. If the water comes up uniformly to the rims of the groove, the instrument is level.

Pythagoras's 3-4-5 Plan

Vitruvius gives an interesting application of this plan in the building of staircases so that the steps may be at proper levels. Suppose the height of the story to be divided into three parts, five of these will give the right length for the stringers of the stairway. Let four parts, each equal to one of the three be set off from the perpendicular, and there fix the lower ends of the stringers.

Hodometer

This instrument, for "telling the number of miles while sitting in a carriage or sailing by sea" is one of the most interesting. The wheels of the carriage were made 4 feet in diameter, so that if a wheel has a mark made upon it and begins to move forward, it will have covered exactly 12 and one-half feet after one revolution. Let a drum with a single tooth projecting beyond the face of its circumference be firmly fastened to the inner side of the hub of the wheel. Then above this, let a case be firmly fastened to the body of the carriage, containing a revolving drum set on edge and mounted on an axle; on the face of the drum there are 400 teeth engaging the tooth of the drum below. The upper drum has, moreover, one tooth standing out farther than the others. Then above, a horizontal drum is placed in another case with its teeth engaging the tooth fixed to the side of the second drum and let as many holes be made in this third drum as will correspond approximately to the number of miles in a day's journey. Let a small round stone be placed in every one of these holes, and in the case containing that drum let one hole be made with a small pipe attached through which when they reach the point, the stones fall one by one into a bronze vessel underneath in the body of the carriage. The result is that the upper drum is carried around once for every 400 revolutions of the lowest and the tooth fixed to its side pushes forward one tooth of the horizontal drum. Since with 400 revolutions of the lowest drum, the upper will revolve once, the progress will be a distance of 400 x $12\frac{1}{2}$ or 5000 feet or one Roman mile.

The same principle was used on ships. The axle passed through the sides of the ship with its ends projecting and four-foot wheels were

mounted on them, with projecting flatboards fastened round their faces and striking the water. The middle of the axle in the middle of the ship carried a drum with one tooth projecting and to this the other drums were attached just as in the land hodometer. When the ship was making headway, the flatboards on the wheel struck against the water and were driven violently back, thus turning the wheels which moved the axle and the axle the drum etc.

Aqueducts

With the exception of the *De Architectura* the only other extant work on Roman engineering is the *De Aquis Urbis Romae* of Frontinus. Frontinus was "curator aquarum" or superintendent of the water supply of the city of Rome appointed under Nerva in A.D. 97. He describes in much detail the materials used and the dimensions of the various aqueducts. The most interesting thing mathematically is the method used by Frontinus for determining the amount of water brought to Rome by the aqueducts, as it affords an interesting illustration of the difficulties of Roman arithmetic. He assumed that the discharge of an aqueduct was equal to the total discharge of a large number of small pipes whose combined cross-sectional areas were equal to the cross-sectional area of the aqueduct, which of course is far from correct. He also failed to take into account the effect of the velocity of the flowing stream on the rate of discharge. Consequently his calculations were of little value. It is estimated that only one half as much water actually was brought to Rome as was stated by Frontinus. Regardless of this, when we read Frontinus's account of the Roman aqueducts we cannot but agree with Pliny: "fatebitur nihil magis mirandum fuisse in toto orbe terrarum."

Bridges

Throughout the Roman dominions, especially during the Imperial period, stone bridges with wide spanning arches of the most massive kind were erected in great numbers. Many were of remarkable size—one bridge over the Acheron being 1000 feet in length—and show in a striking way the skill of the Roman engineers. Many of these bridges still exist in various states of preservation.

One of the most interesting bridges was that described by Julius Caesar (*Bell. Gall.* IV, 17) which was a temporary wooden bridge constructed over the Rhine in the incredibly short space of ten days.

It was supported on a series of double piles, formed by two baulks of timber, each 18 inches square in section, pointed at one end and driven into the bed of the river by machines called *fistucae*. These fistucae, which were also used for ramming down pavements and threshing floors, are supposed to have been very similar to our pile driving machines, which lifted a heavy log of wood shod with iron to a considerable height and then let it fall on the head of the pile. The piles were set in a sloping direction to resist the force of the current and a corresponding row of piles was driven in at a distance of 40 feet, thus forming a wide roadway for the Roman army. The cross-pieces were 2 feet thick and were supported by cross-struts so as to diminish the bearing.

A more permanent military bridge constructed by Trajan across the Danube and designed by the celebrated engineer Apollodorus shows great skill and ingenuity in the way in which he spanned wide spaces with short pieces of timber. According to Gest "it showed the greatness of invincible courage." The piers of this bridge rested upon the foundations made by sinking barges loaded with concrete, foreshadowing rather rudely the modern caisson. There is an interesting story, probably untrue, to the effect that though Hadrian demolished Apollodorus' bridge on the pretense that it might facilitate incursions of barbarians into the Roman provinces, he did it really from jealousy at the success of so great an undertaking.

Tunnels

The Roman engineers, as stated above, were very ingenious. They fitted the lines of their aqueducts to the contour of the ground and selected favorable points for crossing valleys. In building bridges they made up for a lack of theoretical knowledge by an excessive use of materials, and when they drove tunnels they expected the drifts to meet with a fair amount of precision. However, occasionally they did make a mistake. One such mistake is recorded by Gest. Nonius Datus, an engineer of the Third Legion, surveyed and marked the line for a tunnel and then was called away on military duty. During his absence the tunnels were dug, passed the expected meeting point and the drifts were found to have missed each other entirely. Nonius was summoned post haste. When he arrived, he observed in true modern style that as usual they put all the blame on the engineer when it was really all the fault of the contractor, who, in producing the alignment had in both cases deviated to the right. Fortunately the tunnel was for an

aqueduct and not a railroad so Nonius was able to locate a traverse tunnel connecting the two.

Clocks

According to Pliny there was no sundial in Rome until eleven years before the war with Pyrrhus (about 460 A.U.C.), the first being placed by Cursor on the temple of Quirinus. Later the sundials became of very general use and in various forms. Of the numerous kinds, two have been preserved—the hollow hemispherical one and the flat one. The hour lines are in almost every instance engraved in the same manner. They are mostly bounded by the segments of the circle. A midday line *m* is cut by another line running from east to west upon the intersection of which with the hour lines the shadow of the gnomon *g* must fall at fixed times. On these intersecting points the hours are marked. In order to know the hour without giving themselves any trouble, slaves were kept on purpose to watch the "solarium" and "clepsydra" and report each time that an hour expired.

Mart. VIII. 67: Horas quinque puer nondum tibi nunciat, et tu Iam conviva mihi, Caeciliane, venis.

JUVEN. *X. 216:* . . . clamore opus est, ut sentiat auris, Quem dicat venisse puer, quot nunciet horas.

Agrimensores or Gromatici

The Roman surveyors were called Agrimensores—Land Measurers or Gromatici, after the name of their most important instrument. Their business was to measure unassigned lands for the state and ordinary lands for the proprietors and to fix and maintain boundaries. In the case of land surveying the augur looked to the south; for the gods were supposed to be in the north and the augur was considered as looking at the earth in the same way as the gods. Hence the main line in land surveying was drawn from north to south and was called *cardo,* as corresponding to the axis of the world; the line which cut it was termed *decumanus,* because it made the figure of a cross. These two lines were produced to the extremity of the ground which was to be laid out, and parallel to these were drawn other lines, according to the size of the quadrangle required.

The most important instrument of the surveyors was the *groma* which was the prototype of the modern surveyors' cross. The groma is represented on the gravestone of a gromaticus found some years ago at Irvea. Two small planks crossing one another at right angles are supported on a column or post (ferramentum). Plummets are sus-

pended from the planks to guide the operator in securing a vertical position of the column. They were used to guide a surveyor in drawing real or imaginary lines at right angles to one another. They were finally superseded by the "dioptra" which closely resembles the modern theodolite. A treatise on the dioptra by Heron contains a number of propositions illustrative of the use of this instrument. Some of them sound very modern; e.g.,

1. To cut a straight tunnel through a hill from one given point to another.

2. To find the height of an inaccessible point.

3. To sink a shaft which shall meet a horizontal tunnel. (The measurement of angles in degrees was never used by the Greeks or Romans for any but astronomical purposes.)

The most common problem was that of finding the width of a river —"fluminis varatio." One interesting solution proposed by Heron is as follows: *ee* is the accessible shore of a river and *a* is a point on the opposite side. The dioptra is set up at *b* which is a point farther from the bank than the breadth of the river. The sights are directed so that line *ba* shall cut the river at right angles in *h*. The dioptra is then turned through a right angle and the point *c* is taken on *bc* at right angles to *ab*. Then *bc* is bisected at *g* and *gf* is drawn parallel to *bc* meeting *ab* in *d*. A preceding proposition has shown that the sides *ab*, *ac* of the triangle *abc* are bisected in *d* and *f*. Therefore *ad* is equal to *db* and it remains only to measure *db* and *df* and to deduct the measure of *df* from that of *db*. The remainder is the measure of *ah*—the width of the river.

Much more might be said of the practical mathematics of Roman times. Vitruvius, especially, cites many other illustrations of the skill and knowledge of the Romans in architecture and engineering, yet no more need be said to show that "they were resolute in what they undertook, they were good thinkers, they were born engineers and builders."

Bibliography

W. A. Becker. *Gallus* or *Roman Scenes of the Time of Augustus.*
Frontinus. *De Aquis Urbis Romæ,* translation by Herschel (1913).
A. P. Gest. *Engineering.*
Sanford. *A Short History of Mathematics.*
Smith. *Dictionary of Greek and Roman Antiquities.*
Vitruvius. *De Architectura,* translation by M. H. Morgan (1914).
Dictionary of Classical Antiquities.

The Eternal Triangle

A PLAYLET

By Gerald Raftery, *Elizabeth, New Jersey*

Cast of Characters: Three boys, herein referred to as First, Second, and Third.

Properties: Small chair, blackboard, chalk which first boy carries in his pocket, mathematics book with protractor which he carries in his hand, two books and foot-rule which second boy carries (if stage is large a yard-stick may be necessary and can be placed near the black board), book containing small sheet of paper which third boy carries.

(*Discovered,* First Boy *center Rear, reading a book.* Second Boy *enters, Right, whistling and swinging two books and foot-rule. Walks on a few steps, notices* First Boy*; stops whistling and walks on watching him; walks past looking over shoulder. Stops just before exitting and turns back; starts whistling again and walks back past him watching him closely.* First Boy *reads on, turning page of book.* Second Boy *approaches curiously and looks over reader's shoulder.* First Boy *reads on.* Second Boy *stares at reader, then book, then reader again, and at last passes his open hand, palm, slowly between reader's eyes and book, as if to see if he is conscious.*

First (*looking up angrily*). What's the matter? Are you sick?

Second. No. *(Pauses, startled).* I was just wondering what was the matter with you. That—that's a math book, isn't it?

First (*pugnaciously*). Well, what about it?

At this point, Third Boy *enters, and watches the two with interest, drifting closer to the pair until he is behind* Second Boy *when his* (Third Boy's) *turn comes to speak.*

Second (*hesitantly*). You—you weren't reading it, were you?

First (*angrily*). Yes. What about it?

Second. Well—I—you see—uh—well, I was wondering why?

First. What do you mean—why?

Second (*embarrassedly*). Well, uh, haven't you anything else to do?

First (*still angry*). I suppose I could go around snooping over peoples' shoulders and waving my hand in front of their faces.

Second. Well, don't get angry! I mean do you have to read it?

First (*loudly*). NO; I don't have to read it. I'm reading it because I like to read it.

SECOND *(steps back and touches his chin with his forefinger; speaks as though unbelieving and seeking re-assurance).* You like to read it?

FIRST (*firmly*). I like to read it.

THIRD *(comes up quietly and from arm's length taps* SECOND BOY *on the shoulder, who turns;* THIRD BOY *half-whispers).* He likes to read it? (SECOND BOY *nods dazedly.*)

There is a pause while both turn and face the audience, THIRD BOY *with hands sunk deep in his pockets,* SECOND BOY *scratching his head; they both stare at the footlights and shake their heads sadly, then repeat together—*

BOTH (*in a soft voice*). He likes to read it!

FIRST BOY *turns back to his book while the other two confer for a moment in unheard whispers, glancing at him frequently. They seem to come to a decision and after urging the* SECOND BOY, *the* THIRD BOY *approaches* FIRST BOY *and taps him on the shoulder, starting slightly as he looks up.*

THIRD. Say, we know what you're trying to do. You want us to read that book; like Tom Sawyer when he had to whitewash a fence. He made believe he liked it, and then all the other boys wanted to do it.

SECOND (*adds, before third boy has quite finished*). Yes; sure; you can't fool us.

FIRST (*indignantly*). I *don't* want you to read it. Go on away and don't bother me. *(Goes back to his book in disgust, then looks up again to inquire sarcastically.)* Didn't either of you fellows ever read a book?

SECOND. Sure I did.

THIRD. Of course; why I go to the library twice a week.

FIRST. I mean don't you ever read a school-book. Or do you just carry them around so people won't think you work for a living?

NOTE: *The next four speeches are to be spoken in rapid succession, the players almost running them together.*

SECOND. Say, are you trying to be funny?

THIRD. Of course I read them.

SECOND. Lots of times I read them.

THIRD. Sometimes in study periods, when I haven't anything else to do, I read them.

SECOND *(Pauses, as though about to tell something scarcely credible).* Sometimes I even read them when I'm home.

FIRST. Well, then, what are you gaping at me for?

SECOND. But—but—(*stutters helplessly; turns to third boy and gestures impotently at the first.*)

THIRD (*bursts out*). But that's a *math* book!

FIRST (*rises, bangs book down upon chair and clenches fists, placing them on his hips; he stares menacingly at the other two*). *WELL!*

The Second *and* Third Boys *start back alarmed. There is a pause.*

SECOND (*apologetically*). Now if it was, say, history, that would be different.

THIRD. Or maybe geography. (*Proffers the book in his hand.*) They're sort of interesting, kind of, sometimes.

SECOND. Yes; all about (*waves his hand vaguely*) er-ah continents and things, and—ah administrations. *(He appears quite proud of the words.)*

THIRD (*nods his head in vigorous agreement.*) There you are. *(He points to the books.)* There's something you can get a hold on. Hemispheres and wars and things.

SECOND (*also nodding agreement*). Yes *sir! (Makes a contemptuous gesture toward the math book.)* That stuff! All about triangles and such.

THIRD (*levels an argumentative finger at the first boy.*) Did you ever see a triangle?

SECOND BOY *nods and smiles, holding his coat lapel with one hand and rocking back on his heels in an attitude suggesting that the first boy must be quite vanquished by the last remark.*

FIRST (*laughs*). Did you ever see a hemisphere or an administration? And triangles? (*gestures widely*) Why I can see them wherever I want to!

SECOND *and* THIRD boys *start back, frightened, and fall together, clutching each other's shoulders.*

BOTH. Wha—what!

FIRST (*laughs*). I mean I can imagine them anywhere I want to. When you walk straight toward something, don't you just imagine a straight line? Well, that how I see triangles.

SECOND (*sighs, very much relieved*). Oh *(Pauses and swallows).* I see.

THIRD (*loosens collar briefly with one finger*). I thought you were crazy or something.

FIRST. I suppose you think anybody who reads a math book is crazy. (*Turns away impatiently.*)

SECOND (*with alarmed politeness*). No-o, no-o, nothing like that. You see (*very soothingly*) we were just sort of wondering, kind of.

THIRD. Yes, we were wondering just what you liked about it.

FIRST. Well, it (*hesitates*) it's interesting.

SECOND (*surprised*). Interesting?

THIRD. How?

FIRST. Why, if you know all about triangles you can tell how far things are away from you, and all that.

NOTE: *The next five speeches are to be spoken in rapid succession as the speakers move nearer to the first boy.*

SECOND (*surprised*). You can?

THIRD. You mean a trick, like?

SECOND. With your eyes shut, can you?

THIRD. Where does it tell you that?

SECOND. I never found anything like that.

(They both reach for the math book.)

FIRST (*impatiently*). No, it's not a trick. (*Pulls math book away from them.*) And you have to keep your eyes open.

SECOND (*skeptically*). Aw, I bet it's a trick.

THIRD (*gestures contemptuously*). Sure it is.

FIRST (*patiently*). I tell you, it's just a matter of knowing how.

SECOND. Well, you'll have to show me.

THIRD (*shaking his head emphatically*). Yes *sir!*

FIRST. All right. Do you see that corner over there? (*Reference may be made, instead to a pillar, clock, door, or other object.*)

SECOND. Of course I see it.

THIRD (*expectantly*). How far away is it?

FIRST. Guess. (*Then when they both stare at him, he addresses the second body particularly.*) Go on, take a guess.

THIRD (*pats his shoulder encouragingly*). Go on, guess.

SECOND (*folds his arms, puts his head on one side and squints at the corner*). Oh, I'd say about a hundred, er, and fifty, or uh, maybe two hundred, or ——

FIRST (*mimicking*).—or maybe three hundred and fifty or six hundred.

SECOND (*angrily*). All right, you don't have to be so wise about it!

THIRD (*sympathetically*). No; why my scoutmaster says that judging distance ——

FIRST (*interrupting*). Are you a boy scout?

THIRD. Sure I am.

FIRST (*turning to second boy*). And you?

SECOND (*nodding vigorously*). Uh-huh.

FIRST. All right. (*Goes over to a certain spot; looks at his feet and then at the corner.*) Pace me off ten yards in that direction; both of you do it, and check each other. (*Marks the spot with chalk which he takes from his pocket.*)

Pulls a foot-rule from the second boy's books and goes to the black board where he marks off a ten-inch line.

NOTE: *The line on the stage is to be drawn in such a fashion that it will be the base of a triangle of which the indicated corner, post or other object will be the apex.*

SECOND. Say, his ten yards comes to there and mine comes to here. We can't both be right.

FIRST. You're probably both wrong. Just take a spot half-way between the two. (*He goes over to where they stand and marks the spot with chalk.*) Now throw me over that math book. (*Takes book and removes protractor, measuring the angle.*) You see that line on the wall up there; that's ten inches long and each inch of it represents a yard of this line you fellows paced off (*Goes over and adds angle to one end of the chalk-line, and goes on to the other mark on the floor.*)

SECOND. Say, let's see that thing you've got.

THIRD. What is it? Some trick thing?

FIRST (*returning to the board after measuring other angle.*) Don't be silly. It's just a protractor. (*Adds second angle to triangle and closes it up.*) Now watch this. The triangle here is just a little picture of the big triangle that I imagined from here (*points to the floor*) to the corner over there (*points*). It's a scale drawing, and that means that each inch here represents a yard out there.

SECOND. Oh, a scale drawing! Sure, I think I saw something about them in my Scouts' Handbook once.

THIRD. Yes, sure. Where it tells about map-making——

FIRST. Is that one of the books you read sometimes in study periods when you haven't anything else to do?

SECOND. Well . . . yes. (THIRD BOY *nods but is rather embarrassed.*)

FIRST (*sarcastically*). What do you do? Look at the pretty pictures?

THIRD. Aw, hey, don't ride us! We're no geniuses; we don't pretend to be.

SECOND. That's right.

FIRST. Do you have to be a genius to do a little thing like that: finding the distance to that corner?

SECOND. Well, how do we know? You haven't finished it yet.

THIRD. No; go ahead and show us.

FIRST (*incredulously*). What? I haven't finished it? Why all you have to do now is measure one of the side lines. (*Slowly and patiently he explains.*) That triangle there (*points to it*) is a little picture (*measures a space with his hands*) you know, like a picture in a book, picture, picture—of this (*points*) triangle on the ground.

SECOND (*inquiringly*). Well?

THIRD. Yeah?

FIRST (*stares hopefully at their faces but shakes his head sadly at their puzzled look and continues*). Every inch in that triangle (*points to it and then goes over and touches it*) triangle, triangle—represents a yard on the ground, ground (*stoops and pats the floor*) ground.

The second and third boys stare a moment at the first boy, then break into a slow grin.

BOTH. O-oh . . . ye-ah . . . I—see.

Under cover of laughter from the audience they start to explain to each other, going over to the board.

NOTE: *The following two speeches are to be repeated simultaneously to get the effect of jumbled and excited conversation, and may be spoken twice, in whole or in part.*

SECOND. Sure, each inch stands for a yard. Don't you see that? You count the inches and then turn them into yards. Why it's easy; inches stand for yards.

THIRD. Why, of course. Each inch in one of these lines means a yard on the ground. Sure; this is just a little picture, like. You just take yards for inches.

After a pause, the second boy picks up a rule, foot or yard, and measures the distance.

SECOND. Then the corner is —— yards. Gee; that's easy.

THIRD. (*walks away a few steps, staring at the ground and rubbing the back of his head; after a short pause, then:*) Now that's all very well, but wouldn't it be almost as easy to measure the distance, or pace it, or something?

FIRST. Don't be silly. Suppose it was on water, or over a river or a quarry. Or maybe the triangle would be a couple of miles long.

SECOND. Could you really measure a long distance like that?

FIRST. Well, you could get a pretty good idea of it this way. But to get it exactly, that is, right down to an inch or so, you'd have to use a surveyor's transit.

THIRD. Oh, I know what they are. They're those things that look like a telescope on three legs.

SECOND. And you can measure a couple of miles right down to the inch with them? I didn't know they were so exact.

FIRST (*gestures with his hand*) Sure they are. Why there's a certain kind of transit gun that's so darned exact that they have a fellow standing around all the time just holding an umbrella over it to keep the sun from expanding the different parts.

SECOND. Aw, I don't believe that!

THIRD. That sounds to me like so much ——

FIRST (*interrupting vehemently*) It's a fact. My brother worked for the Park Commission one summer, and he told me he saw it.

SECOND. Well, maybe that's because those things can tell heights, too. Maybe that's why they're so delicate.

THIRD. Say, you can't tell how high things are. You can't tell that.

FIRST. Who can't? Of course I can!

THIRD (*who has started back in alarm*). Gee, don't get angry. I was just asking you.

SECOND (*looks about in slight confusion*). How—how high is that wall over there? (*Points to side wall of auditorium.*)

FIRST. Got a piece of paper?

THIRD (*takes paper from the book in his hand*). Here you are.

FIRST (*takes paper and folds one corner so as to bisect a right angle*). Now you have to imagine another triangle running up to the top of the wall. (*He squints along the paper, moving back and forth.*) All you have to do is line this up with the top of the wall; and you must remember to keep the bottom parallel to the ground. There, think that's it. (*He stops moving, looks a second, then takes the paper down from his eye.*) Yes; there you are.

SECOND. Where?

THIRD. What?

FIRST. Oh, I forgot; you fellows don't know anything. All you have

to do now is measure the distance to the wall and add my height which is about five feet, and there you are. That will be the height of the wall.

SECOND. Say, that's pretty good. (*Paces off the distance, counting aloud, by threes, and adds the five feet*) —— feet; why you can measure trees or anything that way.

THIRD (*walks in the other direction, scratching his head; there is a pause*). Now that's all very well, but isn't there some easier way?

FIRST. Well, you could use an instrument they call the hypsometer. You just find your distance from the tree or wall, and set the hypsometer. Then you sight it at the top of the wall, and read the height right off the instrument. That works by triangles, too.

SECOND. Gee, you can do a lot with triangles, can't you?

FIRST. You certainly can. You can figure out your location anywhere in the world by using triangles.

SECOND. Is that a fact?

FIRST. It certainly is. That's how ships find where they are out on the ocean.

SECOND. How do they do that?

FIRST. Well, you use an instrument called the sextant. It helps you figure out a great enormous triangle (*this speech to be done slowly and impressively with wide and sweeping gestures*) with the sun at one corner and the horizon at another corner and the ship at the third corner. And that tells you where you are, far out at sea, miles from land.

SECOND (*in an awed voice*). Gosh, that's great!

THIRD (*walks away a few steps, scratching his head*). Now that's all very well, but isn't there some easier way?

The first and second boys make disgusted gestures and exit while the third boy follows.

THIRD. Aw, listen, fellows—I was just ——

CURTAIN

The International Commission on the Teaching of Mathematics

At the meeting of the International Congress of Mathematicians, held at Zürich early in September, Section VII, devoted to the teaching of mathematics, was concerned entirely with the consideration of the work of the International Commission on the Teaching of Mathematics. Two afternoons were given to this section and in particular to the summary of the reports of the various national committees by Professor Gino Loria of Genoa. This report was incomplete, owing to the delay in receiving all of the returns from certain countries. It is expected that it will be finished during the coming winter and that it will appear in print before the summer of 1933. Professor Loria was able, however, to make an interesting report of progress in the important study of the training of teachers of mathematics in the various countries of the world. In order that ample time should be allowed for this purpose, the address of the president was limited to a brief summary of the history of the Commission and to a proposition for a new and important topic for consideration in the next four years.

At the election of the officers and central committee of the new commission, Professor David Eugene Smith presented the names of Professor Hadamard (Paris) as president; Professors Lietzmann (Göttingen), Heegard (Oslo), and Scorza (Italy) as vice presidents, and Professor Fehr (Geneva) as general secretary, and these officers were then duly elected. They were empowered to act as a central committee, to add to their number, and to select members of the various national committees, in cooperation either with the bureaus of education or with the various national associations of teachers as may seem the better plan. The subject of investigation for the next four years was "The Present Trend in the Teaching of Mathematics in the Several Types of Schools in the Various Countries."

At the closing session Professor Smith was unanimously elected Honorary President of the Commission in recognition of the fact that the organization was first proposed by him (Rome Congress, 1908) and of his labors as vice president and president.

The next meeting of the Commission will be held as the Section on Education at the International Congress of Mathematicians which meets at Oslo, Norway, in 1936.

to do now is measure the distance to the wall and add my height which is about five feet, and there you are. That will be the height of the wall.

SECOND. Say, that's pretty good. (*Paces off the distance, counting aloud, by threes, and adds the five feet.*) —— Say, why you can measure trees or anything that way.

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Address by David Eugene Smith

AS RETIRING PRESIDENT OF THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

It is nearly a quarter of a century since, at the Rome Congress, the first International Commission on the Teaching of Mathematics was organized. The Section on the History and Teaching of Mathematics was very fortunate in the selection of a Central Committee of three, empowered to add to its number and to carry on an investigation into the nature of the mathematics in each school year in the countries represented at the Congress.

The president chosen was a man who combined the genius of a mathematician of first rank with a deep interest in the teaching of the subject, not alone in the universities of the world but in the primary and secondary schools as well,—Professor Geheimrath Dr. Felix Klein of Göttingen. The vice president selected was a distinguished contributor in the field of applied mathematics, Sir George Greenhill, F.R.S. The secretary was selected not only as a mathematician but as the editor-in-chief of *L'Enseignement Mathématique,* the leading international journal on the teaching of the subject under consideration.

This central committee represented the Teutonic, the Latin, and the English-speaking countries, and it proceeded to make the Commission even more international and representative of the educational field than was already the case. Furthermore, it established national committees in the various countries represented in the Rome Congress, these committees being in general appointed by the departments of education in the countries concerned.

In the course of the next four years an intensive study of the nature of the mathematics taught in most of the leading countries of the world was carried on, and by the time the Cambridge Congress met in 1912 there had been published a large number of important reports upon the subject under consideration. Eventually some two hundred such reports appeared in print, constituting the first great world survey of the teaching of any major subject of general school instruction.

The importance of this work can hardly be overestimated. Through it each country came to know precisely what the other countries were doing in each year from the primary grades to the university. Summaries were published in various languages, and the schools everywhere began an introspection and a weighing of values which led to a healthy change in many of the branches of mathematical instruction.

At the time of the Cambridge Congress, however, the work of the Commission was by no means completed. Certain countries had not found it possible to carry on the investigation with anything like the care of other countries, and so the Commission continued its efforts, with the expectation of completing its work by the year 1916. The World conditions from 1914 to 1920 were such, however, as not to permit of any meeting of the Commission. Nevertheless the situation in neutral countries was such as to make it possible for the General Secretary to remain in contact with the delegates and to allow for their projects to be carried out. At the Congress at Strassburg in 1920, manifestly not international, it was agreed by such members of the Commission as were present that the times were not favorable to the continuance of its work, and it was decided to consider the Commission as temporarily disbanded. This action was taken, however, with the hope that the work might be continued when the times were more propitious. In particular, an important line of investigation relating to the training of teachers of mathematics had been begun under the leadership of Professor Loria of Genoa, and it was felt that this should be one of the first studies to be made in the future.

When the Congress met in Bologna four years ago the time for establishing a new Commission seemed to have arrived, and the Section on Education appointed the members of a new Central Commission, empowered to add to its numbers. Professor Klein, who had proved himself an ideal leader from 1908 to 1914, had passed away, and so had Sir George Greenhill, the first vice president, and many members of the various national committees. Moreover, world conditions were not yet favorable for any extensive survey of the mathematical field. Countries were concerned with matters political and economical rather than with such subjects as mathematics in the schools. Even now they are only beginning to consider again with care the ultimate values of any special branches of science or of the humanities at large.

For this reason it was somewhat informally agreed that the new

commission should devote its attention to the completion of Professor Loria's report upon the training of teachers of mathematics in the various countries represented in the Bologna Congress.

This work has been carried on as well as the troubled condition of the world has permitted. Hampered by a lack of funds necessary to pay the expenses of national committees and even to pay the necessary expenses of the General Secretary, it has been very difficult to continue the investigation with anything like the success which attended the efforts of two decades ago. Nevertheless much important work has been done, as will be evident from the report of Professor Loria before this section of the Congress.

Two important questions now arise with respect to the future—(1) Shall the work of the Commission be continued? (2) If so, what shall be its nature? Having given the subject much attention during my presidency, and having consulted with the members both by letter and by personal contact at various times, I feel that it was never more important that the work should continue than it is today. It is a mere platitude to say that the world is in the midst of a great revolution—a revolution not by any means confined to governments, but one relating to social conditions everywhere, to educational ideals and practices, to languages and letters, to our concept of history, to religion, to the fine arts, and to our ideals of life in general. In the field of mathematics in our schools this same spirit of unrest is evident, and particularly so in view of the tendency in some countries to democratize all education to the same level for all individuals. Never in our lives has there been such a necessity for defining clearly our purposes in the teaching of mathematics to the various types of individuals, and for considering how these types may successfully be determined. There is no progressive country from which other progressive countries cannot learn, and the experiments being made by each are certain to be of great value to the rest—perhaps in adopting new methods, perhaps in rejecting what seems of too doubtful value to be considered.

As to the nature of a new Commission I believe that we should at once appoint a nucleus of such a body, including as president a man of the type of the late Professor Klein, and one whose interests and standing are both mathematical and educational. To such a man we should offer the presidency. The other offices should also be filled by this section of Congress, but I should hope that the present secretary might be given such further assistance as he may feel to be

necessary in the conduct of the business. Manifestly this Section cannot, in the brief time at its disposal, canvass the whole world, and select with judicious care the future members.

I should therefore leave with the new officers the power of selecting national committees, either in cooperation with the various governments or by inviting national organizations of teachers in the various countries to carry on the studies to be undertaken. The latter plan may well be found to be more effective, under present political and financial conditions. It has the possible advantage that the national societies will be more ready than the Commission to finance the investigations, either by themselves or through appeals to their governments. The manner of appointment and of financing the work may, however, best be left to the Central Committee, for it is probable that the first plan may be the better in some cases and the second plan in others.

As to the subject of investigation, I suggest as very important "The Present Trend in the Teaching of Mathematics in the Several Types of Schools in the Various Countries of the World."

I therefore offer the following resolution—"Resolved that we proceed at the meeting of this Section two days hence to elect a new International Commission on the Teaching of Mathematics, said Commission to report at the Congress to be held at Oslo in 1936; and that this Commission arrange for national committees according to its best judgment."

If, as I anticipate, this shall bring younger men and women into the Commission, while retaining older members where this is thought advisable, and if it shall result in the vigorous prosecution of some such study as the one which I have suggested, it is my hope that this method of procedure will be continued in future Congresses. The teaching of mathematics should never be static; it should be a dynamic force which should result in continued study and in informational quadrennial reports to the Congresses of the future.

Thanking my colleagues and other friends for their generous support during my vice presidency for many years, and during the presidency from which I now definitely retire, I offer the resolution which it has been my privilege to prepare and read.

A Unit of Instruction on Van Dyke's "Fisherman's Luck"*

By Allan Abbott, *Teachers College, Columbia University*

Purposes

1. To enjoy the experience of reading the essay
2. To share this enjoyment with a social group
3. To enjoy vicariously the experience of fishing
4. To share this enjoyment with a social group
5. To create something which shall express these satisfactions

Aims

1. To learn about trout-fishing
2. To learn about other kinds of fishing
3. To learn about the value of fish as food
4. To learn about the economic importance of fish
5. To learn the place of fish in secular and religious history
6. To learn the effect of fish and fishing upon language
7. To learn to manipulate fish—living, dead, and cooked.

Objectives

1. Vocational; opportunities and needs in fishing; is it a blind alley?
2. Wise Use of Leisure: what wise men have fished?
3. Health: food value of fish; vitamins in cod liver oil
4. Home Making: preparation and cooking of fish
5. Social-civic: Fisheries in colonial days; in the Revolution; in connection with arbitration. How we always won.
6. Religious and ethical: Jonah; miraculous draught of fishes; the fish as a religious symbol; keeping Lent. Kindness to fish.

Big Objective

To realize the place of fish in the modern world

Goal

THE FISH-CENTERED SCHOOL

Activities (leading to further activities)

Unit I (fusion with Science). Make and care for an aquarium.

Unit II (fusion with Home Economics). Prepare and serve: Creamed codfish, boiled salmon, fish chowder.

* Reprinted by permission from *School and Society* for May 7, 1932.

Unit III (fusion with Commercial Education). Study the mail-order ads. of Frank E. Davis, and make better ones.

Unit IV (fusion with language). Make a list of such expressions as "poor fish," "gudgeon" (*obs.*), "sucker."

Unit V (fusion with Library work). Cut out all the pictures of fish from books in the library, and paste them in a notebook.

Unit VI (fusion with Handwork). Make a seine of all the string in all your homes (Creative group-project for the entire class through the term).

Unit VII (fusion with Composition). Write a letter to Dr. Van Dyke, presenting to him the seine, scrap-books, chowder, aquarium, etc., and inviting him to address the school.

Dr. Helen M. Walker of Teachers College, Columbia University, upon reading the above unit wrote to Professor Abbot as follows:

MY DEAR PROFESSOR ABBOT,

May I criticize your admirable unit on "Fisherman's Luck" for its lack of mathematics? It grieves me to see you neglect the rich opportunities in that field. For example:

If John can catch 5 fish in an hour and Henry can catch 7, how long will it take them to catch 20 fish if they both work continuously?

If a fish a *inches long weighs* b *pounds, how much will a fish of the same proportions weigh if it is* c *inches long?*

If a fish which can swim 2 miles an hour in still water starts upstream against a current of 1½ miles an hour and if 40 minutes later a launch which can make 15 miles an hour in still water starts after the fish how long will it be before the launch overtakes the fish?

Go to your local fish dealer and ask him to allow you to measure the lengths of all the fish in his store. Make a graph showing the lengths of the fishes.

If A *has a mean record of 10 fish per day for the year and* B *has a mean record of 12 fish per day for the year, what is the standard deviation of this difference and what is the probability that on any given day* B's *catch will exceed* A's?

If A *goes fishing in a lake known to abound in black bass what is the probability that he will bring home nothing but crappies?*

If five persons eat seven fish in one meal, for how many persons will twelve fish make two meals? Ans. 4²⁄₇.

The possibilities of the theme are limitless. I suggest that a complete course in arithmetic could be built around the price of a pretty kettle of fish, and that a fairly substantial course in algebra could be developed around the subject.

What about astronomy? Should not *Pisces* come in for some attention?

With hearty appreciation of your stimulating efforts to make us all fish-conscious.

HELEN M. WALKER

Demonstrative Geometry in the Ninth Year

Joseph B. Orleans
George Washington High School
New York City

In Chapter III on the Mathematics for Years Seven, Eight and Nine, the Report of the National Committee on Mathematical Requirements contains the following paragraph:

"*Demonstrative Geometry.*—The demonstration of a limited number of propositions, with no attempt to limit the number of fundamental assumptions, the principal purpose being to show to the pupil what 'demonstration' means. Many of the geometric facts previously inferred intuitively may be used as the basis upon which the demonstrative work is built. This is not intended to preclude the possibility of giving at a later time rigorous proofs of some of the facts inferred intuitionally. It should be noted that from the strictly logical point of view the attempt to reduce to a minimum the list of axioms, postulates, or assumptions is not at all necessary, and from a pedagogical point of view such an attempt in an elementary course is very undesirable. It is necessary, however, that those propositions which are to be used as the basis of subsequent formal proofs be explicitly listed and their logical significance recognized."

The Tentative Syllabus in Junior High School Mathematics issued in 1928 by the New York State Department of Education suggests under Demonstrative Geometry: "Conception of formal proof. Introduction to demonstrative geometry. (Whether their proof be assumed or not, the three cases of congruent triangles should be made the basis of a number of the very simplest variety of original exercises.)" And in the suggestions to teachers: "If this work is attempted at all in the junior high school, it should come at the end of the ninth year. It should not be forced on pupils whose previous mathematical record has been unsatisfactory. A period of three or four weeks must be reserved for this introductory course, although it is perfectly possible to carry on a review and extension of other mathematical topics at the same time. The main purpose of such a brief introductory course should be at first acquaintance with the method of geometric demonstration. The amount of ground covered is, there-

fore, comparatively immaterial. It may be well to limit the work entirely to five propositions, namely the three laws of congruence of triangles and the two isosceles triangle propositions. If more time is available, the fundamental constructions may be included, with related exercises of appropriate difficulty."

These quotations from two important documents in the field of secondary school mathematics, with no specific detailed suggestions as to the content of a course in demonstrative geometry to consume part of the ninth year, indicate perhaps that the leaders were not in agreement about the content of such a course.

In 1929 the author of this article at the request of Dr. Reeve submitted, as part of a symposium on the subject, an outline of what he thought a unit of demonstrative geometry in the ninth year ought to be. This unit appeared in the Fifth Yearbook of the National Council of Teachers of Mathematics. Since the outline was not based on actual classroom experience with it, the following experiment was undertaken in the George Washington High School, New York City.

In February, 1930, the author selected a group of thirty boys and girls on the basis of the Orleans Algebra Prognosis Test and segregated them into an algebra class to which he taught this unit of demonstrative geometry in addition to the regular course in elementary algebra as outlined in the New York State Syllabus. The class remained with the teacher for the entire year. A second group was formed in the same way in February, 1931, and a third group in September, 1931. The second group was taught by Dr. Emil Post and the third by Miss Veronica Myers. These groups also remained with their teachers for an entire year.

It is important to note that the groups were composed of pupils whose ability in algebra were above the average. The test used as a basis for the selection has a high correlation with achievement in algebra. The pupils were able to complete each topic of the algebra in less time than does a normal class. There was, therefore, ample time for the teaching of the unit of geometry which required from six to eight weeks.

The selected groups had to be used in this experiment, because the author did not feel justified in risking the credit that the pupils might lose for the year of algebra, if the experiment were a failure. With pupils chosen for ability in algebra there was no such risk.

It is also necessary to note that the demonstrative geometry proposed for the ninth year in the Report of the National Committee

presupposes a great deal of informal geometry in the seventh and eighth years. This was not so in the case of the groups in this experiment. The pupils who begin the study of algebra in the George Washington High School come from the elementary grades in which the mathematics consists entirely of arithmetic. It was necessary, therefore, to crowd into the early part of the ninth year all the informal geometry needed for the understanding of the unit of demonstrative geometry.

During the first term of the ninth year the work in geometry comprised about the equivalent of two weeks of introductory material plus that part of the Orleans unit of demonstrative geometry which precedes the section on parallel lines. In the second term the rest of the Orleans unit was taken up, namely, parallel lines and similar triangles.

The introductory work consisted of constructions through which were developed the fundamental concepts and definitions of early geometric thinking; and these, in turn, led to quantitative discussions of figures and so to the axioms as applied to geometric quantities. This development allowed for a certain amount of correlation between the algebra and the geometry, culminating in the same axioms being used both in the solution of equations and in geometric thinking. A certain degree of correlation persisted when the first part of the Orleans unit, dealing with angles, was taken up. However, in the study of congruent triangles, all pretense of correlation was dropped and the class plan consisted in having a period of geometry as soon as the class had gained a full period over the regular algebra class by virtue of its speed.

In the first half of the second term the same plan was followed in the study of parallel lines (though the additional angle work thus made possible allowed for a spurious correlation). However, with the study of similar triangles, a real fusion of the algebra and the geometry took place. For the geometric work was so timed that the pupils were ready for their similar triangles when they were to begin their work in numerical trigonometry. After the necessary common introduction to the two topics, the work took the form of calculation and demonstration separately and again fusion was not possible.

Judged by results, the course had a salutory effect on the members of the groups. In their elementary algebra regents examinations they did at least as well as and in many cases better than they would have done if they had been scattered in the normal classes. The geometry

accomplished was thus a clear gain. In the geometry fully half of each group did A work, while the average was far above the B grade. One of the classes, in answer to a question set before the pupils, expressed their preference for the geometry over the algebra. They found it "easier and more interesting."

It is questionable, in the opinion of the three teachers conducting the experiment, whether the same plan could be followed with a normal ninth year group, from the point of view of the time required. Even if the introductory work has been covered in the seventh and eighth years, the time required by the normal algebra classes for the elementary algebra course makes no allowance for any geometry. The three teachers have had sufficient experience in the teaching of elementary algebra to justify this conclusion.

The teachers also agree that a unit of demonstrative geometry on the whole is foreign to most of the algebra. The one point of contact of the algebra and the geometry in the work on similar triangles and the numerical trigonometry is valuable. A topic like congruent triangles or parallel lines, however, has no place in a year of algebra. The method of procedure requires either separate periods of algebra and geometry throughout the year, or a period of weeks devoted to geometry exclusively at the end of the year after the algebra has been completed.

The experience with the three groups in this experiment leads the teachers to conclude that something can be done with some geometry in the ninth year. The selection of topics, however, is the problem to be solved, if there is to be real fusion of algebra and geometry throughout the year.

At the end of the ninth year each of the groups continued the work of the tenth year with the same teacher. This arrangement furnished an unusual opportunity for further experimentation which will be described in a later article.

The program for the Annual Meeting of the National Council of Teachers of Mathematics will be found on pages 54 to 56 inclusive of the January issue of the *Teacher*. A good attendance at the meeting is anticipated.

The Principle of Fractions in Arithmetic

By H. E. STELSON
Kent State College, Kent, Ohio

OBSERVATIONS made during several years of teaching in college have convinced me that freshmen have less knowledge of fractions than should normally be expected. This deficiency has undoubtedly been a major cause for the failure of freshmen[1] to master either the art of verifying trigonometric identities or that of manipulating algebraic formulas. This lack of ability to work examples in fractions was very forcibly brought to my attention in teaching the content of arithmetic to students who were preparing to teach in elementary schools. Examinations showed that their ability in working fractions had probably deteriorated since they had left the elementary school.

If by chance they arrived at the correct answer they had no idea what process of thinking or rule had been used in securing the result. The student seemed to work each individual example without thinking of the underlying processes that characterize whole classes of such operations. Indeed a very few freshmen could tell what was meant by the principle of fractions or could describe in general terms the method of adding fractions. It appears that teachers have not explained adequately how this simple principle, $a/b = am/bm$ enables one to add fractions by changing to equivalent fractions. This basic principle has too often been overlooked in formulating arbitrary rules for the adding of fractions. Cancellation in multiplication[2] is merely an application of the principle of fractions although it is seldom thought of in this connection. The rationalizing of the denominator in the process of evaluating a radical may be easily understood as another application of this basic principle.

Teachers should carefully explain the general processes of fractions and then show how the drill exercises are examples of these general processes. To drill without this objective is merely memory work and causes the student to acquire mental inertness.

[1] See article by Luella Pressey, "Needs of Freshmen in the Field of Mathematics," *School Science and Mathematics,* Vol. 30, p. 238.

[2] Mr. J. E. Worthington, MATHEMATICS TEACHER, October, 1923, p. 366, advocates omitting this term from arithmetic.

Teaching the Locus Concept in Plane Geometry

By E. B. Woodford
Alderson-Broaddus College, Philippi, West Virginia

After a student has fully mastered a good definition of "locus," and has solved correctly some fundamental locus problems, he will still have a surprisingly large amount of trouble with originals of this type. Feeling the need of a remedy for this situation, this writer has worked out a series of experiments which enable the student to visualize the required locus. He is then able to construct it with straightedge and compasses. The apparatus is intended as an aid in visualizing the locus and not as a method of construction, except in a few cases. The student should be encouraged to solve the problem experimentally only when he feels the need for experimentation.

Classroom results seem to indicate that this method is exceedingly successful, although no elaborate scientific testing has been done to substantiate this indication.

The student may easily obtain the following apparatus:

(1) thumbtacks,
(2) pencils,
(3) string,
(4) a small carpenter's square,
(5) a rectangular box lid (about four by five inches),
(6) a round box lid (about four inches in diameter),
(7) a disc eraser having a hole in the center (diameter of eraser should be about 1½ inches),
(8) a small stick about 2½ inches in length having 1/16 inch hole in the center,
(9) a celluloid right triangle, having 1/16 inch holes bored very close to the corners.

Several loci may be visualized by rolling the circular eraser, with a pencil through the center, along the restricting line. A problem of this type is: "Find the locus of the centers of circles of a given radius which are tangent to a rectangle." The box lid will serve for the rectangle, the locus being traced out by the pencil.

If a pencil "lead" is wedged into a hole very near the outer edge

of the eraser, and a blunt pencil placed in the center, approximations to the cycloid, epicycloid, and hypocycloid, may be drawn. The circular box lid will serve for the circle in drawing the epicycloid and hypocycloid.

The thumbtacks and string serve, of course, for the familiar method of drawing the ellipse.

To draw the locus of the vertices of the right angles whose sides pass through two given points, the thumbtacks may represent the points, and the carpenter's square or the celluloid triangle with a pencil in the vertex, may represent the right angle. If the sides of the square are kept snugly against the tacks while being moved about, the locus of the vertex will be traced out by the pencil. If the given angle is not a right angle, one of the other vertices may be used.

Using the square in such a manner that the sides are always tangent to the round lid gives the locus of the vertices of the right angles whose sides are always tangent to a given circle.

If the stick with a pencil through the center be moved so that both ends always touch the sides of the round lid, the student will visualize a locus of this type. Another problem of this sort is solved by moving the stick so that the ends always touch the sides of the carpenter's square. The locus of the midpoint of the stick is thus drawn.

It is hoped that the above will be of some help to teachers in teaching what is one of the most difficult concepts of plane geometry.

Sir Isaac Newton

Born at Woolsthorpe, Lincolnshire, December 25, 1642
Died in London, March 20, 1727

Newton . . . declared, with all his grand discoveries recent,
That he himself felt "only like a youth
Picking up shells by the great ocean 'truth'."

. . . .

When Newton saw an apple fall, he found
In that slight startle from his contemplation—
'Tis said (for I'll not answer above ground
For any sage's creed or calculation)—
A mode of proving that the earth turn'd round
In a most natural whirl called "gravitation";
And this is the sole mortal who could grapple,
Since Adam, with a fall, or with an apple.

Man fell with apples, and with apples rose,
If this be true; for we must deem the mode
In which Sir Isaac Newton could disclose
Through the then unpaved stars the turnpike road,
A thing to counterbalance human woes:
For ever since, immortal man hath glow'd
With all kinds of mechanics, and full soon
Steam-engines will conduct him to the moon.

—Byron, *Don Juan*, vii, 5 and x, 1 and 2 (1824)

One of Newton's biographers* calls attention to the fact that Newton's life falls into three parts of nearly equal length: a preparatory period closing with his appointment as Lucasian professor at Cambridge, a period of scientific discovery (1669-1696), and the time of his work in the service of the government in London (1696-1727).

The tangible records of his life before he went to Cambridge consist of a notebook, his name carved on a beam in the Kings School in Grantham, and a sundial on the church in Closterworth. His school career does not seem to have been a distinguished one. The notebook shows a wide range of interest, not unlike that of any observant boy.[1] The sundial foreshadows the mechanical ingenuity that was later shown in the making of scientific instruments.

Newton's copy of Barrow's *Euclid,* bought at a county fair, dates from his early days as an undergraduate in Cambridge. But the book was once discarded as a "trifling work" whose statements were self-evident.

It was at Cambridge that Newton became acquainted with the works that caused him to echo the words of Bernard of Chartres, that if he had seen farther than most, it was because he had stood on the shoulders of the giants. Among the giants were Descartes, Wallis, and Barrow.

It should be remembered that in these days, exact science was in a very elementary state. Newton was to complete the work begun more than a hundred years before with the publication of the Copernican hypothesis, and studied and elucidated by Tycho Brahe, Kepler, and Galileo.

For varying periods from 1665 to 1667, the university was closed because of the Great Plague, and Newton, who had recently taken his

* Selig Brodetsky, *Sir Isaac Newton,* London, 1927.

[1] A transcription of this notebook is given in *Isaac Newton* 1642-1727 edited by W. J. Greenstreet, London 1927. A number of quotations appear in *Sir Isaac Newton,* 1727-1927, edited by Frederick Brasch, Baltimore, 1928.

bachelor's degree, spent these enforced holidays at home in the country where he speculated about the forces that operated between the heavenly bodies, studied phenomena of light, and laid the foundations for his work with the binomial theorem and with the calculus.

On his return to Cambridge, he was elected a fellow of Trinity College, and two years later, he was appointed Lucasian professor in the place of his teacher Isaac Barrow who had decided to devote himself to theology. The duties of his professorship gave Newton time for research and provided him with a modest living.

In optics, Newton studied the spectrum, invented the reflecting telescope, investigated refraction, and adopted a corpuscular theory of light instead of the wave theory held by Descartes and Huygens.

It was during the same vacation period that Newton laid the foundations of his work with the binomial theorem. The problem in question was one on which John Wallis was working—to find the area bounded by a curve in the form $y = (1-x^2)^n$ and the x-axis. Wallis was able to find these areas for values of n that are positive integers. By a clever use of the method of interpolation, Newton was able to find the series for the areas when n is fractional. In 1676, he had occasion to refer to his material and it seems to have been at this time that he extended the work to finding the expansion of a negative or a fractional power of a binomial. As given by Newton, the binomial theorem reads:

$$\left(P+\frac{P}{Q}\right)^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n}AQ + \frac{m-n}{2n}BQ + \frac{m-2n}{3n}CQ + \cdots.$$

where A, B, C, . . . are the first, second, third . . . terms of the expansion.

It is significant that this theorem found immediate application in the calculus although the proof for the theorem with fractional exponents was not given until 1774.

Newton's study of gravitation culminated in the publication of his *Philosophiae Naturalis Principia* commonly known as the *Principia* in 1687. The work appeared under the imprimatur of Samuel Pepys, then president of the Royal Society, but although that organization had sanctioned its printing, the expenses of publication and the labor of proof reading were borne by Edmund Halley.

In several instances, Newton's discoveries were claimed by others. Hooke said that the inverse square law (the law of gravitation) was his. Leibniz's followers and perhaps Leibniz himself, claimed that the calculus of Leibniz preceded Newton's fluxions. Unfortunately New-

ton was reluctant to publish his researches and accordingly the precise dates are difficult to establish.

In 1696, Newton was made Warden of the Mint. Three years later, he was appointed as Master of the Mint, a position which carried with it a considerable income and great influence. It also gave him an opportunity to do valuable work in metallurgy. He became president of the Royal Society in 1703 and was knighted by Queen Anne in 1705.

Pope suggested for his epitaph the couplet

> "Nature and Nature's laws lay hid in night.
> God said, 'Let Newton be,' and all was light."

There is a long inscription on his monument in Westminster Abbey, but his grave has simply the line: Hic depositum est Quod Mortale fuit Isaaci Newtoni. VERA SANFORD

Eliakim Hastings Moore

1862-1932

IN THE DEATH of Professor Moore the country lost a man whose influence upon mathematics and its presentation was more pronounced than is generally realized. Graduating at Yale (A.B., 1883; Ph.D., 1885), he pursued his studies in Berlin (1885-86), returning to accept a tutorship at Yale during the next two years followed by assistant professorship for the same length of time. In 1891 he went to Northwestern University as associate professor, and the following year he was called to Chicago as a professor, becoming the head of the department in 1896. He retired from his professorship in 1931.

His early contributions to mathematical journals were in the field of geometry (*Amer. Journ. of Math.*, X, 17-28, 243-257). He then turned his attention to the theory of groups (*Bulletin* of the N. Y. Math. Soc. and the Am. Math. Soc., 1893, 1894; *Proceedings* of the London Math. Soc., vol. 28), the function theory (*Rendiconti* of the Circolo Mat. di Palermo, IV, 186, IX, 86; *Bulletin* of the Am. Math. Soc., I, 252) and to algebra (*ibid.*, III, 372; Math. Annalen, LI, 417). In his later years he devoted himself to general analysis in which field his best mathematical research was done.

While not himself a teacher of note, he did much to call the atten-

tion of mathematicians in this country to the necessity for radically reforming the teaching of the subject. This was done in his address as president of the American Mathematical Society a generation ago, an address which was welcomed by all progressive teachers although its significance was less realized by the mathematical faculties of the universities.

His great success lay in the carrying out in Chicago the ideals which Sylvester had established at Johns Hopkins. It seems to have been largely through his influence that such leaders as Bolza and Maschke were brought from Germany to assist in making Chicago one of the great centers of mathematical research in this country. It is for this that he will have a prominent place in the history of mathematics in America.

David Eugene Smith

Registration at the National Council Meeting

Edwin W. Schreiber, *Secretary*

As the 1933 Annual Meeting of The National Council of Teachers of Mathematics at Minneapolis, Minnesota, approaches, the secretary is very anxious that the members, guests and visitors register properly. Four different colored registration cards are used to classify our registration.

1. *Visitors and guests,* that is *non-members* register on green cards.
2. *Members* who are attending their *first* annual meeting register on white cards.
3. *Members* who have attended more than one meeting but less than half of the annual meetings register on a red card.
4. *Members* who have attended *half or more* of the annual meetings register on a blue card, indicating thereby their "true blue" loyalty to the organization. A member does not continue in the blue class unless he maintains his record of attending at least half of the annual meetings. If he does not measure up he slips back into the red class.

The following list indicates when and where our annual meetings have been held.

1920—Cleveland	1924—Chicago	1930—Atlantic City
1921—Atlantic City	1925—Cincinnati	1931—Detroit
1922—Chicago	1926—Washington	1932—Washington
1923—Cleveland	1927—Dallas	1933—Minneapolis
	1928—Boston	
	1929—Cleveland	

Before you go to Minneapolis will you decide which of the annual meetings you have attended and thus facilitate registration by asking for the proper colored registration card. Persons attending any sessions of *The Council* should register so that the Secretary will have his records complete.

◆ NEWS NOTES ◆

FOLLOWING is the program of the fall meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England, Bulkeley High School, Hartford, Connecticut, October 22, 1932.

"The Two-Factor Theorem of Mental Abilities," Burton H. Camp, Wesleyan University.

"A Mathematics Teacher Seeks Guidance in Guidance," C. Hapgood Parks, Weaver High School, Hartford.

"Have You Ever Taught the College Board Examinations?" Carroll G. Ross, Mount Hermon School.

"Regents' and College Entrance Examinations in Mathematics," W. S. Schlauch, New York University.

"Caelometry," Wm. Fitch Tcheney, Jr., Connecticut Agricultural College.

The officers of this group are: Dorothy S. Wheeler, Bulkeley High School, Hartford, president; H. M. Dadourian, Trinity College, Hartford, vice-president; Arthur D. Platt, Mount Hermon School, secretary; Mary Noyes, High School, Bristol, treasurer; B. H. Brown, Dartmouth College and C. W. Holway, High School, Northampton, directors.

Following is the program of the Southeastern Oklahoma Education Association, Durant, October 28, 1932:

"Meeting the Arithmetical Deficiencies of the First Year Algebra Class," Ruth Caldwell, Haworth.

"My Methods of Instruction in Algebra I," Verdie Goss, Heavener.

"Breaking the Barrier," Vida Loyd, Achille.

There was also an address by J. O. Hassler of the University of Oklahoma and a Round Table discussion led by Allen Berger of Durant.

Walter Rappolee, Madill, is chairman and Vida Loyd, Achille, secretary of the Association.

Following is the program of the Southwestern Oklahoma Education Association, Elk City, Oklahoma, October 21, 1932. Henry Ford, Clinton, is chairman, and Winifred Robbins, Elk City, is secretary of the Association.

"Improvement of the Method of Teaching Mathematics in the Grades with the Aim of Improving the High School Course," H. T. Becker, Altus High School.

"The Value of an Algebra Workbook," Hall Worthington, Frederick High School.

Round Table Discussion led by J. R. Pratt, S.W.T.C.

Review: "Sixth Yearbook of the National Council," Alta L. Carder, Cordell High School.

"The Value of a Geometry Workbook," R. O. Webb, Lawton High School.

Round Table Discussion led by Clarence E. McCormick, S.W.T.C.

"What the High School Teacher Should Give the Students," Dean S. W. Reaves, College of Arts and Sciences, University of Oklahoma.

Following is the program of the East Central Oklahoma Educational Association, Ada, Oklahoma, October 28, 1932.

"The Human Worth of Mathe-

matics," A. M. Wallace, East Central State Teachers College.

"Computation of Logarithms," Mr. Lowe.

"Some Things a High School Teacher Should Emphasize," E. D. Meacham, University of Oklahoma.

The Research Committee of the Men's Mathematics Club of Chicago and the Metropolitan Area consisted of the following members in 1931-32: E. R. Breslich, University of Chicago; Clyde Brown, Senn High School, Chicago; Clark Cell, Skokie Junior High School, Winnetka; W. H. Clark, Crane Junior College; R. R. Cromwell, Austin High School, Chicago; A. S. Hathaway, Northwestern University; J. R. McDonald, Morton Junior College; O. M. Miller, Chicago Normal College; J. A. Nyberg, Hyde Park High School, Chicago; O. E. Overn, Lake View High School, Chicago (chairman); Evert Owen, Oak Park and River Forest Township High School, Oak Park; F. W. Runge, Evanston Township High School, Evanston; O. I. Rusch, Concordia Teachers College, River Forest; H. E. Slaught, University of Chicago; Wm. A. Snyder, New Trier High School, Winnetka.

This committee worked as a whole on the consideration of the general objectives of high school and college mathematics, the specific objectives being in the hands of subcommittees: for the junior high school under R. R. Cromwell; senior high school, F. W. Runge; for the college, W. H. Clark. Another committee under the chairmanship of Mr. Overn dealt with the accumulation, classification, and grading of familiar life situation problems. The officers of the club for 1931-32 were: E. C. Hinkle, president; F. W. Runge, secretary-treasurer; Walter S. Pope, recording secretary; and a program committee composed of W. H. Clark (chairman), J. R. McDonald, and J. J. Urbancek.

The programs of the meetings from October through March were:

October 16, 1931: "Some projects of the research committee," O. E. Overn; "Record keeping on geometry homework," R. F. Briggs; "What shall we teach in ninth grade mathematics?" President Laughlin, Chicago Normal College.

November 29, 1931: "A word concerning the Bulletin of the research committee," O. E. Overn; "Some remarks on curriculum changes as affecting senior high school mathematics," H. D. Merrill; "Some uses of mathematics in economics," Theodore O. Yntema.

December 18, 1931: "A plan of departmental testing," J. R. McDonald; "Practical applications of mathematics to engineering problems," Harry Cole, Architect's Office, Board of Education, Chicago.

January 15, 1932: "Teaching of manipulation by the problem approach method," J. C. Piety; "Some interesting correlations," C. M. Austin; "Measuring the development of functional thinking in mathematics," E. R. Breslich.

February 19, 1932: "A mathematics teacher's library," H. C. Wright; Social aspects of mathematics," J. S. Georges.

March 25, 1932: "The mathematical methods in applied and theoretical physics," Philip A. Constantinides, Crane College.

The program for the April meeting appears in the MATHEMATICS TEACHER for October.

The officers of the Mathematics Club of Columbus, Ohio for 1931-32 were: Metta Philbrick, president; Oakley Wiseman, vice-president; Virginia Titus, secretary-treasurer.

At the November meeting 1931, Miss Marie Gugle spoke on the subject, "What mathematics shall we teach? to whom? and why?" At the February meeting, Mr. James Weaver of Ohio State spoke on the "Present mathematical situation in the light of past experience." In March, the club heard the report of the delegate to the meeting of the National Council of Teachers of Mathematics in Washington.

The officers for the current year are: Oakley Wiseman, 123 Glencoe Road, president; F. A. Sheridan, 60 W. Weber Road, vice-president; Ruth Swinehart, 1495 Michigan Avenue, secretary-treasurer.

The program of the mathematics section of the Ohio Educational Association meeting at Columbus, April 8, 1932, under the chairmanship of Professor R. L. Morton of Ohio University, was as follows:

"Some worthy special objectives for secondary mathematics," Paul C. Recker, Toledo.

"Protecting the social investment in arithmetic by a margin of safety," Carmille Holley, Miami University.

"Mathematics as an interpreter of the social and economic environment," W. S. Schlauch, New York University.

The mathematics section of the Nebraska State Teachers Association District 2 at Omaha, Nebraska, October 28, 1932, has as its president, Arthur L. Hill, State Teachers College, Peru. The secretary is Amanda E. Anderson, Central High School, Omaha. The program was:

"The teaching of geometry in secondary schools," C. A. Huck, State Teachers College, Peru.

"A plea for the ninth grade algebra student," Lorella Ahern, Nebraska City.

"The subject of limits in secondary mathematics," J. M. Earl, Municipal University, Omaha.

"How to make the subject of algebra interesting to beginners," Roxie McGrew, Auburn.

This notice appears in the *Peru Pedagogian,* State Teachers College, Peru, Nebraska:

"Alpha Mu Omega, honorary mathematics fraternity, met Monday evening, October 3, in the mathematics room. The following officers were chosen for the year: president, Virgil Bugbee; vice-president, Nadine Andrews; secretary-treasurer, Ora Ferguson.

"The aim of Alpha Mu Omega is to develop and promote interest in the study of mathematics and to investigate subjects of mathematical interest that are not presented in the classroom. Meetings are held every two weeks when a program is conducted under the leadership of students."

The Northeastern Ohio Teachers Association met on October 28-29, 1932. The program of the mathematics section was under the chairmanship of Guy Wright, Youngstown. The following papers were presented:

"Helping to reduce the number of failures in ninth year algebra," Wesley A. Soule, Youngstown.

"Another experiment in large classes," Dessie M. Post, Cleveland.

"Motivating geometry," Harry L. Geis, Youngstown.

"Trigonometry in high school," Harmon C. Welsh, Youngstown.

"High School mathematics from the college viewpoint," J. L. Jones, Akron University.

The Northwestern Ohio Teachers Association met at Toledo on the same dates. The mathematics program was under the leadership of C. A. Robbins, Findlay, Ohio. The program consisted of a discussion of illustrative materials from the history of mathematics for junior and senior high school use by Vera Sanford, School of Education, Western Reserve University.

The Cleveland Mathematics Club had the following officers in 1931-32: W. O. Smith, South High School, president; Cora Lederer, Central High School, vice-president; Margaret Debnrock, Fowler Junior High School, secretary; W. N. Rutledge, Lincoln High School, treasurer; Florence B. Miller, Fairmount Junior High School, chairman of the executive committee.

On December 1, 1931, Miss Ida M. Baker of the School of Education, Western Reserve University, addressed the club on the use of radio in the teaching of mathematics. Dr. E. R. Hedrick of the University of California at Los Angeles spoke at a dinner meeting February 17. On March 21, Mr. A. Brown Miller presented his report of the Annual Meeting of the Council to which he was a delegate. On April 19, Mr. Charles H. Lake, Assistant Superintendent of Schools, addressed the club on "Budgets." On May 27, Mr. William Betz was the speaker at the annual meeting of the club.

The officers for 1932-33 are: A. Brown Miller, Fairmount Junior High School, president; Elizabeth Thomas, Patrick Henry Junior High School, vice-president; Helen E. Baldwin, John Hay High School, secretary; H. C. Seasholes, John Adams High School, treasurer; J. J. Rush, West High School, chairman of the executive committee.

Professor W. D. Reeve was the speaker at the first meeting of the club for the current year.

The High School Conference Division of the National Council of Teachers of Mathematics was called to order by the chairman, Miss Gertrude Hendrix, in room 300, Mathematics Building at 9:07 A.M., November 18, 1932.

Mr. William Clark of Champaign High School presented a paper, "Objectives of Plane Geometry."

A round-table discussion ensued. Those contributing were Mr. E. W. Schreiber of Macomb and Mr. H. C. Wright of Deerfield Shields, Highland Park.

A letter to the organizations affiliated with the National Council of Teachers of Mathematics was read by the chairman. This letter was from Mr. William Betz of Rochester, New York. It was a report concerning the formation of: 1. The Committee on Individual Differences; 2. The Committee on Geometry; 3. The Committee on Policy.

The chairman appointed the following committee to nominate one member to replace the retiring member of the program committee of the High School Conference and a delegate and an alternate to the Minneapolis meeting of the National Council: Dr. Lytle, University of Illinois, chairman; Miss Nelson, Urbana High School; Mr. C. N. Mills, State Normal University. Mr. E. W. Schreiber of Macomb spoke on

membership in the National Council and of the yearbook of the National Council.

Miss Beulah Shoesmith of Hyde Park High School presented a paper, "An Ounce of Prevention." This was followed by a round-table discussion on "objective tests." Those participating were: Mr. E. W. Schreiber, Miss Martha Hildebrant, Mr. C. N. Mills, Mr. H. C. Wright, and Dr. E. B. Lytle. The meeting adjourned at 11:40 A.M. and was called to order again at 2:00 P.M., Miss Gertrude Hendrix, presiding.

A report on the nominating committee was made by Dr. Lytle. The committee nominated Miss Martha Hildebrant of Proviso Township High School, Maywood, for the vacancy on the program committee; Prof. E. H. Taylor for the delegate to the National Council Meeting at Minneapolis; and Prof. C. N. Mills, alternate. This left the following committee: Prof. J. T. Johnson, Chicago Normal College, chairman; Mr. Wm. Clark, Champaign High School, vice-chairman; Miss Martha Hildebrant, Proviso T.H.S. secretary.

There was a motion that the report be declared an election. There was a second and the report was declared an election. Prof. A. R. Crathorne, Professor of Mathematics, University of Illinois, presented a paper "The Problem of Mathematical Statistics."

Miss Martha Hildebrant spoke on membership in the National Council and on the yearbooks of the National Council.

Mr. J. T. Johnson, Professor of Mathematics, Chicago Normal College, presented a paper "Adapting Instructional Material to Individual Differences in Learning."

This was followed by a round-table discussion. Those participating were Dr. E. B. Lytle, Mr. E. W. Schreiber, Mr. Mesingkamp, and Mr. H. C. Wright.

The meeting adjourned at 4:15 P.M.

WILLIAM CLARK, *Secretary*

The Winthrop College Branch of The National Council of Teachers of Mathematics was organized November 10, 11, 1932, with a membership of twenty. The following officers were elected: President, Miss Alice Ann Grant; vice-president, Miss Kate Rosen; secretary-treasurer, Miss Worthe Roland; local editor for THE MATHEMATICS TEACHER, Miss Cam Rhodes Rawlinson.

Those appointed to committes are as follows:

Committee on Constitution and By-Laws: Dr. G. T. Pugh, Miss Fannie Watkins, Miss Louise Weill.

Program Committee: Miss Fannie Watkins, Dr. G. T. Pugh, Miss Hortense C. Rogers, Mr. W. B. Nichols, Miss Lillian Henderson, Miss Cam Rhodes Rawlinson.

Publicity Committee: Miss Kate Rosen, Miss Elizabeth Harmon, Miss Willie Faye Taylor, Miss A. Roberta Wooten.

Membership Committee: Miss Louise Weill, Miss Catharine Allgood, Miss Elizabeth Jones.

Social Committee: Miss Mildred Stukes, Miss Nelle Douglas, Miss Worthe Rowland.

Committee on Absences and Excuses: Miss Sarah Mae Hester, Miss Thelma Robinson, Miss Carolyn Snipes, Miss Willie Faye Taylor.

According to the Constitution, the president is a member, ex-officio, of all committees; the vice-president is a member of the Program Committee and the secretary-treasurer is a member of

the Absence and Excuse Committee.

The first regular meeting was held on November 15, 1932. The program consisted of an inauguration speech by the president, Miss Alice Ann Grant, a part of which consisted of "Fifteen Fundamentals for Successful Teaching," formulated by Miss Grant and presented from the platform by the student members. Dr. Pugh, Miss Watkins and Miss Rogers discussed and made suggestions concerning the Fundamentals for Successful Teaching.

Following is the membership list of the Winthrop College Branch of the National Council of Teachers of Mathematics:

Catharine Allgood, Nelle Douglas, Alice Ann Grant, Elizabeth Harmon, Lillian Henderson, Sarah Mae Hester, A. Elizabeth Jones, W. B. Nichols, Dr. Griffith T. Pugh, Cam Rhodes Rawlinson, Thelma Robinson, Hortense C. Rogers, Kate Rosen, Worthe Rowland, Carolyn Snipes, Mildred Stukes, Willie Faye Taylor, Fannie Watkins, Louise Weill, Annie Roberta Wooten.

CAM RHODES RAWLINSON,
Local Editor

A conference of the alumni of Kent State College, Kent, Ohio, who are teaching mathematics in high schools in northeast Ohio was held at Kent December 31, 1931. The morning program consisted of talks on the mathematics clubs and extra-curricular activities of the mathematics teacher. The afternoon program was composed of talks on the requirements in mathematics necessary for high school graduation. The following forty-four teachers were present:

A. C. Bahler, Sugar Creek; Pauline Berg, Wellsville; E. B. Bloom, Garrettsville; G. B. Bowman, Mantua; W. Care, E. Canton; G. H. Chapman, Kent; C. L. Cook, Kent; Lillian Davis, Wellsville; W. W. Davis, Rootstown; G. M. Dewell, Scienceville; E. B. Goddard, Canton; F. N. Harsh, Kent; M. Hercheck, Atwater; A. C. Heritage, Ravenna; Effie Hysell, Elyria; Bess Kauffman, Kent; Wm. King, N. Canton; J. Koeppe, Aurora; C. F. Koontz, Welshfield; F. A. Lawrence, Cleveland; Bonnie McClung, Wadworth; W. D. McConnell, Lowellville; R. E. Manchester, Kent; H. S. Martin, Youngstown; Margaret Mills, New Lyme; Mrs. Alice Murlin, Plymouth; J. C. Murlin, Plymouth; L. E. Laragon, Kenton; Pearl Porter, Hudson; D. C. Price, Kent; W. L. Richey, Scienceville; J. T. Reid, Cleveland; H. P. Rogers, Kent; Neda Schaeffer, Kent; H. W. Short, Brady Lake; P. E. Spacht, Mantua; H. E. Stelson, Kent; Eleanor Stone, Canton; W. E. Suter, Cleveland; H. L. Vine, Canton; E. D. Walter, Warren; C. L. Weaver, Cuyahoga Falls; C. E. Williamson, Roscoe; H. C. Winkler, Kent.

NEW BOOKS

Plane Geometry. By Elizabeth Cowley. Pages xii+368. Silver, Burdett and Company, 1932. Price $1.40.

This book omits such technical terms as ***antecedent, consequent, equivalent, homologous, perigon, scalene, subtend,*** and ***trapezium*** in order to decrease the vocabulary burden of the student.

Definitions are given only when they are needed and such simple terms as ***point, straight line,*** and *angle* are not defined at all but are discussed.

The student is introduced to the study of geometry through a study of geometry in nature and in the modern world.

The text meets the requirements of the various extra-mural boards and the theorems are arranged so as to meet the needs of groups of varying ability.

Arithmetic for Teachers. By Harriet E. Glagier. Pages xv+291. McGraw-Hill Book Company, 1932. Price $2.00.

It is the purpose of this book to provide for the future teacher of arithmetic a connected idea of the subject matter of arithmetic: the concept of number; the development of a number system; the four fundamental operations with natural numbers, or integers, those numbers at which we arrive by the simple process of counting; the fractional or artificial number in its various forms, the common fractions, the decimal fraction, the per cent, and the extension of the four fundamental operations to include them; the fundamental idea of a unit of measure, and the endless field of measurement; as well as those notions of space—length, surface, volume—which were taught so long as mensuration but today are approached as intuitive geometry. The book is planned to cover a semester's work of three recitations per week. It has been used in syllabus form in that way prior to publication.

Standard Service Algebra. By G. M. Ruch and F. B. Knight. Pages xv+528. Scott, Foresman and Company, 1932. Price $1.32.

Standard Service Algebra is a learning instrument as well as a drill and problem book. About 40% of the total space is devoted to the thorough, step-by-step development of new topics, intended to lead the pupil gradually into full insight of the logic of algebra rather than to leave him with merely the capacity to juggle symbols.

Mastery of the formula and equation is regarded as the basic task of first-year algebra. The test of this mastery is the ability to solve practical problems. To this end more than twelve hundred concrete problems are provided, a fourth of them in the 20 standardized, graded Problem Scales.

The author claims that the effective maintenance program is a noteworthy improvement over contemporary texts. Twenty-five standardized *Self-Testing Drills* provide periodic review of all fundamental operations. A graphic record of accomplishment on these Drills is provided through the Individual Progress Charts. Systematic chapter and semester reviews are included.

This new textbook is a major unit

in the *Standard Mathematical Service,* an organized program for all grades.

The new Day Arithmetic. By Fletcher Durell, Harry O. Gillet, and Thomas J. Durell. Pages x+260 (approx). Chas. E. Merrill Company, 1931. Books 3 to 6 inclusive, 68c. Books 7 and 8, 72c.

This is one of the newer series of arithmetics the purpose of which is to abolish incomplete learning inaccuracy and uncertainty and bring about mastery of the fundamentals. This is done by providing among other things:

1. Language easily understood by the pupil.
2. Teaching and practice material prepared in terms of both the pupil and the subject.
3. Economy particularly in learning.
4. For the wide range in abilities among pupils.

Solid Geometry. By Herbert E. Hawkes, Mr. A. Luby, and Frank C. Touton. Pages xvi+216. Ginn and Company, 1932. Price $1.24.

This is a new edition of a previous one in which the authors have been guided by their experience in teaching from the earlier text. A few definitions have been rephrased; many exercises have been regraded; the order of theorem has been changed in a few places in an attempt to improve the order; the numerical exercises have been modified; a reasonable amount of new-type examination material has been added for review and examination purposes; and certain theorems have been rewritten in the interest of greater simplicity and clearness.

Simplified Geometry. By C. V. Durell and C. O. Tuckey. Pages x+280. G. Bell and Sons, Ltd., London, 1931. Price if issued in three parts 1s.6d. each. Complete book in one volume 4s.

The subject matter of this book has been selected and arranged to meet the needs of junior students in England, but it is also suitable for other pupils in secondary schools and technical schools of various types.

Part I is a complete commentary on the contents of a box of instruments; and its scope is defined precisely by that theme.

Part II contains a systematic development of geometrical properties of informal lines.

Part III lays more emphasis on the presentation of formal proofs and standard constructions.

This book should be of interest to American teachers of geometry.

Arithmetic for Today. By Robert F. Anderson and George N. Cade. Three book series. Silver Burdett and Company, 1931. Price 72c each.

It is the aim of this series to provide a course the successful pursuance of which will give the pupil such knowledge of number and such skill in the processes with number as will enable him to compute accurately and readily and to apply his knowledge and skill to the solution of problems sufficient in number and so graded as to meet his present interests and future needs.

Matriculation Trigonometry. By C. V. Durell. Pages viii+151. G. Bell and Sons, Ltd., London, 1932. Price 3s.6d.

The object of this new book is to provide a concise course in trigonometry which will be suitable for matriculation and certificate candidates in English schools. It will be of interest to those American Teachers whose needs are somewhat similar.

Advanced Algebra. Vol. I. By C. V. Durell. Pages viii+193. G. Bell and Sons, Ltd., London, 1932. Price 4s.

This book is the first volume of an "Advanced Algebra," which will complete in a second volume the school course for mathematical specialists in England.

In accordance with modern practice, calculus methods have been employed whenever the treatment of the subject is simplified thereby. The treatment of limits and convergence (in Chap. IV) is confined to what is suitable for a first reading.

Non-Interpolating Logarithms, Cologarithms and Antilogarithms. By F. Johnson. The Simplified Series Publishing Company. San Francisco, 1931.

The book contains ten complete tables none of which require any interpolation or calculation of any kind. These tables include (1) five-place logarithms of all five-place numbers, (2) five-place antilogarithms of all five-place logarithms, (3) four-place logarithms of all four-place numbers, (4) four-place cologarithms of all four-place numbers, and (5) four-place antilogarithms of all four-place logarithms. Besides these principal tables there are others of interest.

Plane Trigonometry with Tables. By William Wilder Burton. Pages x +125. Thomas Y. Crowell Co. Price $2.50.

This is a new book intended for use in liberal arts colleges, universities, and engineering schools. The author claims to have developed the course over a period of years and as a result of experience with several different classes. He has, therefore, departed somewhat from the modern tendency of concise treatment and has planned the course so as to stimulate self reliance on the part of the student.

Teachers of trigonometry will be interested in looking over another attempt to simplify the traditional course in trigonometry for teaching purposes.

Text and Tests in Plane Geometry. David Eugene Smith, William David Reeve and Edward L. Morss. Ginn and Company, 1933. 286 pp. List price $0.88, subject to the usual discount.

This work is so far removed from the usual text book in geometry that it merits a longer description than is offered in the usual book notes, but while the consideration of its features makes one enthusiastic about it, the real "proof of the pudding" will come when it meets the test of class-room use.

While the book is best adapted for the ideal scheme in which each pupil is issued a new book each year, the authors show in the preface that it may also be used in classes where one set of books must last for several years. In the latter case, the student's notebook replaces the text book for written work.

The new features of this geometry begin with the cover which shows a bit of shore line set with Greek temples on the one side and, on the other, the sky-scrapers of a great American city. The pictures might be on the jacket of a book of travel. It suggests at once a contact between the present day and the past.

Another novelty is the page size which is that of a workbook, and which accordingly makes the book weightier than its number of pages suggests. The table of contents shows a division into "units." This has the advantage of dignifying the introductory material and of giving the student the sense of accomplishment. It should obviate the

sense of hurry often given to the latter part of the course when Books III, IV, and V are considered in less time than has been devoted to Book I. The units have the titles: Informal Geometry, Approach to Demonstration, Congruence, Parallelograms, Loci, Circles, Inequalities, Proportion and Similarity, Ratio in Trigonometry, Areas of Polygons, Circle Measurement. At the close of each unit there is a page of review or test material and at the close of the book there are twenty-seven tests on the eleven units and five on a general review. These pages are perforated, and the implication is that many teachers will remove them at the beginning of the term for use as examinations.

The authors have kept to a unit page organization in that each page is an attack on one particular point noted in its heading, and, unless the page consists of problems for practice or for review, it has a discussion addressed to the pupil, the development of a piece of theory, and *something for the pupil to do*. This arrangement is informal but it is clear and one suspects that the student will have a fair idea of what it is all about.

The units are illustrated with half tones that are excellently reproduced. In fact, they are far above the standard of those generally used.

The introductory unit considers the origin of geometry, geometry in nature, in architecture, geometry around us, what you study in geometry, classification of geometric figures, construction examples, and the idea of congruence. Review exercises appear at intervals in each unit. A valuable feature of the construction theorems is that instead of compressing the matter into a single diagram, there is a cut representing the given conditions, another showing the construction lines, and, when a proof is given as is the case after Unit I, a diagram shorn of construction lines for the proof. The workbook feature is prominent in the exercises that follow the constructions. By the close of the first unit, the student has done a certain amount of experimentation, but the authors caution him that better methods will be given him later and he is warned not to place too much confidence in inferences.

In Unit 2, definitions, axioms, postulates of procedure, and postulates of fact are developed as bases of proof. Directions are given for framing a geometric proof. In this unit and in succeeding ones, thirty-one basal theorems are given in full and are designated by an index. (☞) The theorems are limited to those on "approved lists" and except for the basal ones, they are given in skeleton form and the student is expected to supply the missing parts which may be the plan of proof, or the reasons for the steps. Corollaries are put with the originals but are shown in bold faced type.

The authors give no reason for their use of the equal-rotation-definition of parallel lines, but this procedure allows them to introduce theorems regarding parallel lines immediately after the one on vertical angles and it allows them to postpone the much discussed superposition proofs for congruence theorems. It is likely that the side-angle-side theorem carries more conviction when it comes as Proposition 14 than when it is Proposition 2. This order also permits the proof of the converse of the *pons asinorum* by a direct method.

The later units follow the pattern of units 3 and 4,—a quantity of originals, frequent reviews, and a review test. As we would expect, the ideas of continuity are illustrated by series of drawings marked Geometric "Moving Pictures."

The exercises are carefully graded, and they include some interesting new illustrations as for instance, finding the depth of the Greenland ice pack (p. 191).

Perhaps no better illustration of the way in which the book is addressed to the pupil can be given than by quoting the introduction to the first work with limits. It has been the reviewer's experience that competent pupils reject the work that is often given on this subject as allowing them to do things that their earlier work did not permit. They have characterized the text as giving a "slipshod" proof. In this case, the authors of the *Text and Tests* state the matter frankly. The preceding page has given an illustration of constant, variable, and limit. Then (p. 239) we read:

"In the case of a regular polygon inscribed in a circle, we have another example of the jumping frog. In this case, however, we have an increasing apothem and a decreasing side.

"No satisfactory proof involving limits can be given in elementary geometry. A rigorous proof depends upon more advanced mathematics, particularly the calculus.

"We can, however, discuss the next two theorems in a way that will be as convincing as a formal proof. We shall organize the discussion by steps, as in previous propositions, but it should be clearly understood that such a discussion does not constitute a regular proof. Following each discussion, we shall simply assume that the theorem is true."

At first sight, the reader suspects that this text is best fitted for the slower pupil, but closer study will convince him that there are many more originals than he at first supposed, and that they are so arranged as to develop power on the part of the more able students.

VERA SANFORD

Analytic Mechanics. By Joseph B. Reynolds. Pages x + 345. Prentice-Hall, Inc., 1929. Price $4.00.

This text in Analytic Mechanics is intended primarily for students in technical schools and colleges. Many practical applications are given and explained, while others are suggested or involved in the problems set for the student to solve. The development, however, will be found to be such that, with some omissions, the text can be used equally well where a course in pure or theoretical mechanics is desired. In kinematics, the foundation required is a working knowledge of analytic geometry and the calculus. In kinetics, a working knowledge of elementary mechanics and kinematics is required.

Teachers of the subject, with years of experience, have discovered that the principles of any branch of mathematics become a part of one's mental equipment only after frequent applications to the solution of problems. This is particularly true of the subject of mechanics in which there are comparatively few principles and many and varied applications. The exercises set for the student, therefore, comprise one of the most important parts of a text on the subject. With this conviction, the author has put into this text a large number (about 700) of carefully graded exercises; most of which are original, for the student to solve. About one fourth as many illustrative examples are solved and explained in order to demonstrate the method of applying the principles developed. It is believed that these exercises, on the whole, will not be found too difficult for any student prepared for the study of analytic mechanics; yet a few will test the powers of the brightest student. Care has been exercised to adjust the data, when possible, so that the student will not be lost

in a maze of arithmetic work thus losing sight of the principles involved.

This book is exceptionally well suited to follow "Elementary Mechanics" by the same author. These two books, when combined, provide teachers with an ideal tie-up of material for class use. In Reynold's "Elementary Mechanics," for example, is offered a text which does not require calculus, while, on the other hand, "Analytic Mechanics," covering as it does the advanced work, requires a knowledge of calculus.

Algebra For Today. By Wm. Betz. Second Course. Pages x + 502. Ginn and Company, 1931. Price $1.36.

This book like the author's first course is an attempt to make algebra more meaningful to young pupils. The course is intended to be flexible enough to satisfy the varying needs of different schools or groups of pupils. Many new features have been included which will be of interest to all teachers of intermediate algebra.

Help the National Council Grow!

We hope that our readers will study the following table which gives the increases and decreases in the various states and foreign countries from 1929 to 1933.

	1929	1933		1929	1933
Alabama	51	31	Nevada	2	7
Arizona	12	6	New Hampshire	20	27
Arkansas	31	19	New Jersey	157	202
California	169	151	New Mexico	10	11
Colorado	65	50	New York	599	608
Connecticut	88	82	North Carolina	100	67
Delaware	3	13	North Dakota	8	11
Florida	33	29	Ohio	267	227
Georgia	27	34	Oklahoma	95	63
Idaho	15	6	Oregon	23	16
Illinois	357	295	Pennsylvania	401	446
Indiana	156	91	Rhode Island	35	35
Iowa	156	93	South Carolina	28	37
Kansas	184	122	South Dakota	19	15
Kentucky	29	37	Tennessee	41	46
Louisiana	35	21	Texas	146	115
Maine	33	28	Utah	10	10
Maryland	71	68	Vermont	18	18
Massachusetts	278	217	Virginia	63	65
Michigan	231	138	Washington	36	34
Minnesota	119	90	West Virginia	44	45
Mississippi	47	46	Wisconsin	127	112
Missouri	89	82	Wyoming	16	9
Montana	12	12	Philippine Islands	12	42
Nebraska	103	79	Foreign Countries	107	111

In order to help us in getting more efficient results will you please do the following two things:

1. Send for as many of the blanks like those on page 61 as you can use to advantage in obtaining new members either through canvas of your individual friends and colleagues or through group meetings with which you are connected in the future. In the case of the latter it is rather essential that someone at such meetings give a spirited talk on the work of the National Council of Teachers of Mathematics and the importance of each individual teacher being a member.

2. Give us the correct and permanent address of new members, at least the one where they wish the magazine to be sent. It makes no difference to us whether it is the school or home address, but we would like to be sure which is preferable in order that no issue will be delayed or missed.